Jurnal
Teknologi

Full Paper

THE COMPARISON OF EMPIRICAL FORMULA TO PREDICT THE INCIPIENT MOTION OF WEAK COHESIVE SEDIMENT MIXTURE

Mojtaba Porhemmat, Wan Hanna Melini Wan Mohtar*, Chuah Run En, Juliana Abd Jalil

Department of Civil and Structural Engineering, Faculty of Engineering and Built Environment, Universiti Kebangsaan Malaysia, Bangi, 43600, Selangor, Malaysia

Article history
Received
10 March 2016
Received in revised form
13 May 2016
Accepted
25 May 2016

*Corresponding author
hanna@ukm.edu.my

Abstract

Available incipient sediment motion models were mostly developed based on homogeneous non-cohesive sediment. In non-homogeneous sediment mixture, particularly with fraction of cohesive particles might have an effect on the values of incipient sediment motion using the available models. In this study the effects of weak cohesive material on the incipient motion of sand-silt mixture were investigated in laboratory and compared with different existing formulas. Fine sand with median grain size of d_{50}=153 μm was used whilst the cohesive material is kaolinite-silt with d_{50}=28 μm. The sand-silt mixtures were prepared with sand percentage (*Sa*)/Silt percentage (*Si*) distributions ranging from 100*Sa*/0*Si* to 0*Sa*/100*Si*. The condition for incipient sediment motion was defined in categories number 4 (i.e. frequent particle movement at nearly all locations) and number 6 (i.e. permanent particle movement at all locations) of 7 categories introduced by the Delft Hydraulic. Values of critical Shields parameter were monotonously increased as the percentage of silt increased but reached a relatively constant value as the silt percentage reached 50%. Comparing the data for the particular sediment range used in this study proved that the Brownlie, Van Rijn and Miedema relationships are capable to calculate the Shields parameter within the range of category 4 and 6.

Keywords: Incipient of motion, sand-silt mixture, cohesive sediment.

Abstrak

Pergerakan ambang sedimen selalunya dihasilkan berdasarkan sedimen homogen tidak jelekit. Di dalam campuran sedimen tidak homogen, pecahan partikel jelekit mungkin mempunyai kesan ke atas nilai pergerakan peringkat ambang sedimen dengan penggunaan model yang sedia ada. Kajian ini melihat kesan bahan jelekit lemah ke atas pergerakan peringkat ambang campuran tanah berpasir-kelodak dikaji di dalam makmal dan dibandingkan dengan formula empirical sedia ada. Pasir halus dengan saiz median d_{50}=153 μm telah digunakan manakala bahan jelekit yang digunakan ialah kaolinite-kelodak dengan saiz d_{50}=28 μm. Campuran pasir-kelodak disediakan dengan peratusan (*Sa*)/Silt peratusan (*Si*) pula di dalam julat 100*Sa*/0*Si* hingga 0*Sa*/100*Si*. Definisi bagi pergerakan ambang ditakrifkan di dalam kategori nombor 4 (i.e. kekerapan pergerakan zarah di hampir semua tempat) dan nombor 6 (i.e. pergerakan zarah kekal ke semua tempat) daripada 7 kategori yang diperkenalkan oleh Delft Hydraulic. Nilai parameter kritikal Shields meningkat secara sekata dengan peningkatan peratusan kelodak tetapi mencapai nilai malar pada campuran 50% kelodak. Perbandingan data dengan kajian lepas membuktikan hubungkait Brownlie, Van Rijin dan Miedema mampu untuk mengira parameter Shields di antara julat dalam kategori 4 dan 6.

Kata kunci: Pergerakan peringkat awal, campuran pasir-kelodak, sedimen-lekit.

1.0 INTRODUCTION

One of the most important factors related to sediment transport and is extensively studied by different researchers is threshold motion. The threshold motion happens when the flow velocity exceeds its critical value, which the sediment particle starts to move [1]. Threshold motion is important in the study of sediment transport, channel degradation, and stable channel design and other hydraulic issues [2,3,4,5,6]. Despite the numerous work, it is not easy to clearly define at what flow condition a sediment particle will start to move due to the stochastic nature of sediment transport along the sedimentary bed. Most of the studies defined threshold motion based on own definition and observation, where terms such as initial motion, several grain moving, weak movement, and critical movement were commonly used [7,8].

The most common presentation used to characterise threshold motion is using the Shields' Diagram. Shields (1936) used the result of studies done by Gilbert (1914), Kramer (1932) and Cassie (1935) and produced the most used curve for predicting threshold motion in sediment transport [2,9,7,10]. However, the diagram was developed based on homogenous non-cohesive sediment whereas natural sediment normally contain the cohesive material such as clay and silt.

While hydrodynamics and erosion/transport mechanisms of non-cohesive sediments are well understood, erosional properties of cohesive sediments have proved more difficult to describe and quantify. Clay-size particles, mostly platy-shaped, provide the cohesive nature of river beds, and the cohesiveness is highly dependent on the soil structure [11]. Stability of the sediment bed in the range of sand size and coarsest, following the gravity forces while for cohesive range, the inter particle electrochemical forces has an important role [12]. The cohesion factor of clay makes its behaviour different with sand. When a sand mixture has percentages of clay, not only the gravity forces contribute to the stability, the effect of the inter particle electrochemical forces should too be considered.

This study investigates the threshold criteria of sand-silt mixture in a laboratory with different percentage of sand and silt. The sand-silt mixture was varied from 100/0 to 0/100 percentage of both sediment type.

2.0 METHODOLOGY

The experimental recirculating Perspex made flume with dimensions of 0.5 × 0.5 × 10 m^3 was used in this study. The working area with dimensions of 0.6 × 0.15 m^2 located inside the flume, 4.4 m from upstream. The flow velocity (u) and water depth (y) were controlled by the valve located in the flume inlet and the tailgate located in flume downstream respectively. Figure 1 illustrates the schematic of the flume setup.

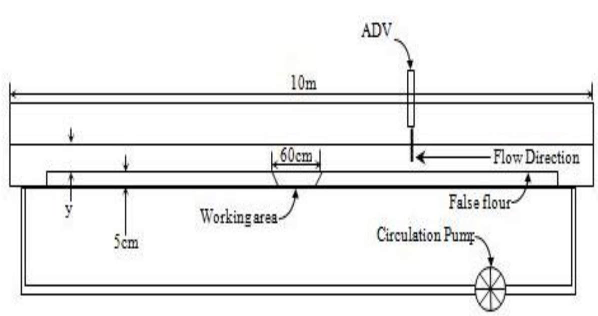

Figure 1 The schematic experimental setup

The ADV (Acoustic Doppler Velocimeter) used to measure the flow velocity. The ADV was located in the upstream of the working area and measured the velocity 7 cm from the streambed. A vernier scale was used as a point gauge to measure the depth of water. The sediment motion was visually observed based on two definitions of initial motion and weak motion.

The sediment layer depth in the working area was set at 5 cm following the false floor located in the upstream and downstream of the working area. The sand-silt mixture with different percentages of the kaolinite (0-100%) with d_{50}=28.2 μm as the cohesive material and fine sand with d_{50}=152.7 μm were used in this experiment.

Separate series of experiments were done to find the plastic (PL) and liquid limits (LL) for the cohesive kaolinite-silt material. The result shows that the plastic limit is 23.1 and the liquid limit is 34.9. Plasticity index (LL) was calculated using equation 1 and equalled to 11.7 which means that the cohesive material is in the range of slightly plasticity.

$$PI = LL - PL \tag{1}$$

The gradation parameter (σg) where used to study the uniformity of the sediment mixture, describe as

$$\sigma_g = \sqrt{\frac{d_{84}}{d_{16}}} \tag{2}$$

where d_{84} and d_{16} denote the sediment size at the 84% and 16% cumulative percentile, respectively.

The result of particle size analysis shows σg for sand is 1.73 and for kaolinite is 2.95. Low σg of 1.73 indicates that the sand used is uniformly distributed sediment. For kaolinite, a rather high σg of 2.95 suggests that the cohesive material has large distribution of sizes. This is supported by the detailed analysis of size distributions, where the percentages of d<=4 μm is 6.94, 4 μm <d<=63 μm is 72.03 and d>63 μm is 21.03, and as most of the kaolinite material falls under the 4 μm <d<=63 μm region, the kaolinite used here can be denoted as silt.

To prepare the sand-silt mixture, the dry sand and silt were mixed in a mechanical mixer. After mixing in dry condition, water (around 25%) was added to the mixture to produce a mixture within the range of plastic limit (23.1-34.9). Then the process of mixing was continued until the mixture became completely homogenous. Mixtures of sand-silt were prepared with varying percentages of sand/silt as 100/0, 80/20, 60/40, 50/50, 40/60, 20/80, and 0/100. Table 1 shows the particle size parameters for sand-silt mixture. The median grain size d_{50} was obviously decreased when the cohesive materials were added to the sample.

The mixture was added to the working area in few layers up to 5 cm, whereby adding each layer, it was compacted by compactor 25 blows per 10 cm diameter to ensure the uniform compaction.

The prepared sediment bed was scraped by a scraper to ensure a flat bed. Flow was slowly (u<5 cm/s) introduced into the flume to minimise any undesirable disturbance to the bed. When the flow depth reached about 7 cm, and believed to have negligible effect of sheet flow on the bed, then the flow velocity was gradually increased to see at which velocity particles start to move nearly in all working area frequently (category 4), which described in Table 2. Then the velocity was continually increased to observe the permanent particle movement in all working area (category 6). The critical velocities represent the velocity of motion in two different categories 4 and 6 were denoted as $u_{c1}$and u_{c2}, respectively.

Table 1 The particle size parameters for different percentages of kaolinite

	The percentages of kaolinite-silt (S_i)						
Particle size parameters	100	80	60	50	40	20	0
d_{10} (μm)	5.6	6.1	6.6	7.8	7.9	10.9	60.1
d_{16} (μm)	8.6	9.5	10.3	11.8	12.5	17.4	82.2
d_{50} (μm)	28.2	33.3	39.3	47.9	55.3	99.9	152.7
d_{84} (μm)	74.8	107.9	137.1	168.4	174.5	212.0	246.4
d_{90} (μm)	95.3	145.1	176.4	206.8	211.3	246.0	275.6
σg(gradation parameter)	2.9	3.4	3.6	3.8	3.7	3.5	1.7

Table 2 The definition of seven levels of erosion described by the Delft Hydraulic, Breusures (1972), as cited by Miedema, [13]

1	Occasional particle movement at some locations.
2	Frequent particle movement at some locations.
3	Frequent particle movement at many locations.
4	Frequent particle movement at nearly all locations.
5	Frequent particle movement at all locations.
6	Permanent particle movement at all locations.
7	General transport (initiation of ripples).

The initial motion was describe using the Shields parameter (θ_c) define as:

$$\theta_c = \frac{u_*^2}{(S-1)g.d_{50}} \tag{3}$$

where u_* is the critical shear velocity, S is ρ_s/ρ , ρ_s is the sediment density, ρ is the density of water, g is the gravitational acceleration.

The critical shear velocity was calculated using the following formula:

$$u_* = \frac{0.4u_c}{ln\left(\frac{y}{d_{50}}\right)} \tag{4}$$

where u_c is the critical velocity and used for both u_{c1} and u_{c2}.

The original shields diagram is not really practical as both axis use shear velocity u_* and it is usually an unknown parameter, whereby making it an implicit graph[14]. To make it explicit, the graph was converted to other axis system, where the dimensionless grain dimeter D_* is often used. The dimensionless grain diameter is also called Bonnelive parameter as cited by Miedemea [13] and can be calculated as

$$D_* = d\sqrt[3]{\frac{R_d.g}{v^2}}, \tag{5}$$

where d is the grain diameter, R_d is relative density (specific gravity), g is gravitational constant and v is the kinematic viscosity. Relative density has the same value as specific gravity (ρ_s/ρ).

By placement of R_d, g and v in the equation (5) for water we can approximate D_* which will be almost 20 times of d in mm.

$$D_* = 20.d \tag{6}$$

Shields did not derive a model or an equation, but published his finding as a graph [6]. For practical purpose many researchers revisit the Shields curve and represent it in a more simplistic empirical equation to obtain the incipient sediment motion value

[15,16,17,18,13]. The different equations to approximate shields parameter can be seen in Table 3. The performance of each equation was evaluated using RSME parameter. Each calculated value was compared with the observed data for categories 4 and 6 denotes as θ_{c1} and θ_{c2} , respectively

Table 3 **I**ncipient sediment motion equations used in comparison analysis, the calculate RMSE values for each equation is included

Equations	Relationship	RMSE (compared to category 4)	RMSE (compared to category 6)
Soulsby& Whitehouse (1997)	$\theta_c = \frac{0.3}{1+1.2D_*} + 0.055 \times (1 - e^{-0.02D_*})$	0.068	0.018
Brownlie (1981)	$\theta_c = \frac{0.22}{D_*^{0.9}} + 0.06 \times 10^{-7.7D_*^{-0.9}}$	0.026	0.025
Van Rijn (1993)	$\theta_c = \frac{0.24}{D_*}$ $D_* < 4.5$ $\theta_c = \frac{0.14}{D_*^{0.64}}$ $4.5 < D_* < 10.2$ $\theta_c = \frac{0.04}{D_*^{0.1}}$ $10.2 < D_* < 17.9$ $\theta_c = 0.013D_*^{0.29}$ $17.9 < D_* < 145$ $\theta_c = 0.055$ $145 < D_*$	0.051	0.012
Zanke (2003)	$\theta_c = \frac{0.145}{D_*^{0.5}} + 0.045 \times 10^{-1100D_*^{-2.25}}$	0.06	0.108
Miedema (2008)	$\theta_c = \frac{0.2285}{D_*^{1.02}} + 0.0575(1 - e^{-0.0225D_*})$	0.041	0.013

3.0 RESULTS AND DISCUSSION

Figure 2 illustrates the effect of silt percentage on the critical average velocity. Increasing the percentage of cohesive material were seen to consistently increase the critical average velocity, until the silt percentage was at 50%. Above this percentage, the critical velocity remains relatively constant at $u_{c1} \approx 25.7$ and $u_{c2} \approx 29$ cm/s, despite the increased content of kaolinite.

To investigate the obtained profile with regards to the established Shields diagram, Figure 3 shows the critical Shields parameter for both category 4 (θ_1) and category 6 (θ_2) data against the particle Reynolds number (R_p). Particle Reynolds number (R_p) was calculated as the flow parameter

$$R_p = \frac{u_* d_{50}}{v} \qquad (7)$$

where v is the kinematic viscosity.

The Shields curve was plotted (as solid line in Figure 3), which was redrafted from [19]. Note that all data was in the hydraulically smooth region, indicated by R_p< 5.

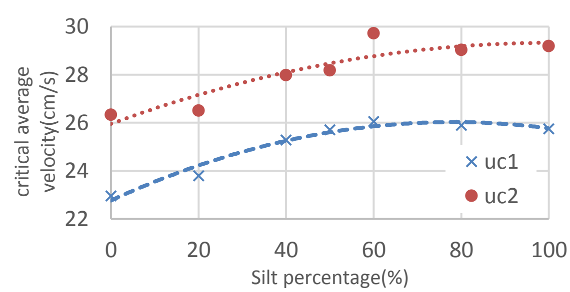

Figure 2 The critical average velocity against the percentage of Kaolinite-silt material

As the plot shows, with increasing percentage of cohesive material, the Shields parameter was monotously increased. As describe in Table 1, higher percentage of kaolinite material in the mixture resulted in lower d_{50}. The initial motion graph falls closely on the Shields diagram and the weak motion profile was found to be slightly above the Shields diagram. Data observed in both categories 4 and 6 of incipient motion showed a similar profile

For particles with Reynolds number greater than 0.6, Shields diagram is underestimated and for particles with Reynolds number smaller than 0.6, the critical Shields parameter is overestimated, which is in agreement with Mantz [20]. The sediment in the range of $0.02 < d_{50} < 0.16$ mm was used in this study.

Using Equation (5), the Bonneville parameter was found in the range of $0.5 < D_* < 3.1$, which is in the hydraulically smooth region. By placement of the Bonneville parameter in the relationships presented in Table 3, the calculated Shields parameter for each equation were shown in Figure 4, along with the measured θ_1 and θ_2 profiles.

The Brownlie, Van Rijn and Miedema formulas were seen to fall closely within the θ_{c1} and θ_{c2} lines. The

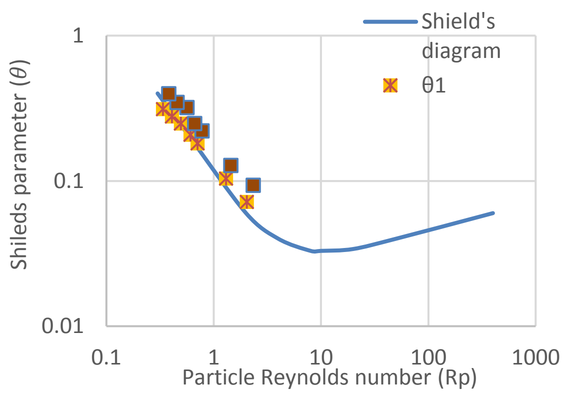

Figure 3 Critical Shields parameter against particle Reynolds number for categories 4 and 6 of motions defined in Table 2

Zanke relationship, however underestimated the Shields parameter in the range of $0.5 < D_* < 2$, but as the particle size increases (i.e. $D_* > 2$), the Shields profile was found laying between the θ_{c1} and θ_{c2} range. The Soulsby & Whitehouse relationship was evidently underestimate the Shields parameter in this particular range of sediment. Among the equations discussed here, the Brownlie approximation was found to have the minimum error ($RMSE = 0.026$) with the θ_{c1} profile whereas Van Rijn has the minimum error ($RMSE = 0.012$) when compared to θ_{c2}. The interesting point is that the value calculated by the Brownlie equation is almost matched with the average value of θ_{c1} and θ_{c2} with $RMSE = 0.004$.

The Soulsby & Whitehouse and Zanke relationships have the highest error with both θ_{c1} and θ_{c2}. The Soulbsy & Whitehouse equation has the error of $RMSE = 0.068$ and $RMSE = 0.118$ while Zanke has the error of $RMSE = 0.06$ and $RMSE = 0.108$ with θ_{c1} and θ_{c2} , respectively.

The difference in calculated Shields parameter for each equation become evident for fine grain size, particularly when $D_* < 2$, the Zanke and Soulsby & Whitehouse curves deviate from the rest of the profiles as the particle size becomes smaller .

4.0 CONCLUSION

The critical velocity was monotonously increased as the percentage of silt increased but reached a relatively constant value when the silt percentage reached 50%. The incipient motion data agrees with the Shields' diagram at the hydraulically smooth region when characterised using both definitions as

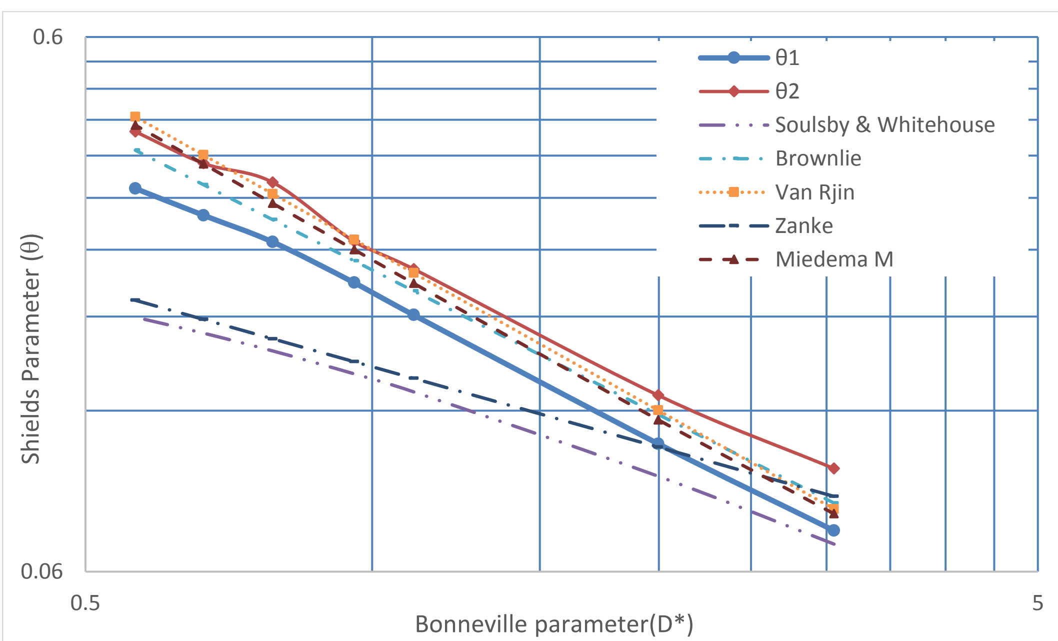

Figure 4 Critical Shields parameter against Bonneville parameter (D_*). Thick and thin solid line represents observed data in categories 4 and 6 (θ_{c1} and θ_{c2}), respectively

category 4 and category 6. It can be said that the cohesion of this range of silt does not play a significant role in the threshold criteria of incipient sediment motion but the median diameter of particle (d_{50}) play the main role in this particular range of sediment size.

Different relationships were used to approximate the Shields parameter. In the particular range of particle size used in this study (i.e. $0.5 < D_* < 2$), Van Rijn and Miedema relationships approximate the Shields parameter with the lowest error for category 6 (Permanent particle movement at all locations). Based on this data, both Zanke and Soulbsy & Whitehouse formulas have the highest RMSE when compared to both θ_{c1} and θ_{c2} particularly for $D_* < 2$. This indicates that for very small particle size using these two equations might give inaccurate prediction for incipient sediment motion.

Acknowledgement

The authors are grateful for the financial support given by the Ministry of Education, Malaysia under the ERGS/1/2013/TK03/UKM/02/7 grant and the Ministry of Science, Technology and Innovation for the funded 06-01-02-SF1077 grant.

References

[1] Vanoni, V. A. 1975. Sedimentation Engineering. *ASCE*, New York.

[2] Shields, A. F. 1936. Application of Similarity Principles and Turbulence Research in Bed-Load Movement. *US Dept of Agr. Soil Conservation Service Cooperative Library, California Institute of Technology, Pasadena, Calif*.

[3] Buffington, J. M. and Montgomery, D.R. 1977. A Systematic Analysis of Eight Decades of Incipient Motion Studies, with Special Reference to Gravel-Bedded Rivers. *Water Res.* 33(8): 1993–2029.

[4] Ye, Z. 2010. Erosion Threshold and Erosion Rate of Seabed Sediments. PhD thesis. The University of Western Australia, Aust.

[5] Wan Mohtar H. 2011. The Interaction between Oscillating-Grid Turbulence and a Sediment Layer. PhD thesis. The University of Nottingham, UK.

[6] Debnath K. 2012. Local Scour around Non-Circular Piers in Clay-Sand Mixed Cohesive Sediment Beds. *Engineering Geology*. 151: 1-14.

[7] Kramer, H. 1935. Sand Mixtures and Sand Movement in Fluvial Models. *Trans. ASCE.* 100(1): 798–878.

[8] Yang C. T. 2006. Erosion and Sedimentation Manual. *U.S. Department of the Interior Bureau of Reclamation, Colorado, USA*. *Chap* I, 1-5.

[9] Gilbert, K.G. 1914. The Transportation of Debris by Running Waters, *U.S. Geological Survey Professional Paper* 86.

[10] Cassie, H.J. 1935. Uber Gesclziehebewegung, Preuss. Versuchsanst Fur Wasserbau Und Schifibau,Berlin, Mitt., vol. 19 (translation on file at the U.S. Soil Conservation Service, Washington, D.C.).

[11] Wang Y. C. 2013. Effects of Physical Properties and Rheological Characteristics on Critical Shear Stress of Fine Sediments. PhD thesis. Georgia Institute of Technology, USA.

[12] Sanford, L. P. 2008. Modelling a Dynamically Varying Mixed Sediment Bed with Erosion, Deposition, Bioturbation, Consolidation, and Armouring. *Computer & Geoscience*. 34(10): 1263-1283.

[13] Miedema, S. A. 2008. An Analytical Method to Determine Scour. *WEDA XXVIII & Texas A&M*. 39(1): 8–11.

[14] Miedema, S. A. 2010. Constructing the Shields Curve, a New Theoretical Approach and its Applications. *WODCON XIX, Beijing China*. (September).

[15] Brownlie, W. R. 1982. Prediction of Flow Depth and Sediment Discharge in Open Channels. PhD thesis, California Institute of Technology, USA.

[16] Soulsby, R., & Whitehouse, R. 1997. Threshold of Sediment Motion in Coastal Environments. Pacific Coasts and Ports '97: Proceedings of the *13th Australasian Coastal and Ocean Engineering Conference and the 6th Australasian Port and Harbour Conference*. 1: 145-154.

[17] Zanke, U. 2003. On The Influence of Turbulence on the Initiation of Sediment Motion. *International Journal of Sediment Research*. 18(1): 17–31.

[18] Van Rijn, L. C. 1993. Principles of Fluid Flow and Surface Waves in Rivers, Estuaries, Seas and Oceans. *Aqua Publication*, (Edition).

[19] Rouse, H. 1939. An Analysis of Sediment Transportation in the Light of Fluid Turbulence, Soil Conservation Service Report no. SCS-TP-25, U.S. Department of Agriculture, Washington, D.C.

[20] Mantz, P. A. 1977. Incipient Transport of Fine Grains and Flakes by Fluid-Extended Shields Diagram. *Hydraulics division ASCE*. 103(6): 601-615.