THE ROLE OF HIGH DEGREE POTENTIAL COEFFICIENTS AND SATELLITE ALTIMETER DATA IN GRAVITY FIELD APROXIMATION IN THE MALAYSIAN REGION

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Synopsis

Earth Gravity models (OSU81, OSU86E, and F) defined by a set of high degree potential coefficients were used to generate the geopotential geoid in the Malaysian region. In the very near future, land gravity measurements can be carried out where the station positioning in the survey will be by Global Positioning System (GPS) operating in differential mode. In areas with scarce height benchmark, especially in the remote areas of Peninsular Malaysia, the geopotential geoid can be utilized in conjunction with the satellite derived ellipsoidal heights to yield the orthometric heights of the gravity stations.

Satellite altimeter data has the ability to provide high frequency gravity field information in the surrounding marine areas. The method of gravity anomaly recovery in the Tioman test area was based on the theory of least squares collocation. Gravity anomaly maps derived from satellite altimeter data can be used to scan large off-shore areas for detecting density contrasts within the oceanic's outer crust, and thus providing an indirect indication of potential hydrocarbon deposits.

Introduction

Gravity data is collected on land using gravimeters in conjunction with known reference-base stations for which values of gravity are usually given in the International Gravity Standardization Net 1971 (IGSN71). The process that lead to the establishment of a gravity base network in Peninsular Malaysia has been documented in Majid (1987a). However, additional work is needed so that progress can be made towards the creation of a *national gravity data base* for geodetic and geophysical purposes. The geographic distribution of the existing gravity measurements remains mostly in the coastal areas, while in many hinterland areas, gravity data is simply not available because of physical inaccessibility problem. Of equal importance is the gravity data over the surrounding marine areas. The planned national gravity data base should include gravity information from both land and the surrounding ocean areas.

Since the current holding of terrestrial-based measurements is lacking, this paper explores two alternative techniques, namely, the use of Earth Gravity Models and satellite altimetry technique for the gravity field approximations in the Malaysian region.

The Role of the Earth Gravity Models

Malaysian Geopotential Geoid

Geoid has been loosely defined as the equipotential surface of the earth's gravity field which would coincide with the mean sea level if the latter were undisturbed and affected only by the earth's gravity field (National Geodetic Survey, 1986). The need to know the geoid was originally driven by the need to reduce geodetic measurements from the surface of the earth to the reference ellipsoid on which computations were to be made and geodetic coordinates to be given. With the advent of the Global Positioning System (GPS) there has been a renewed interest in the geoid undulation N (i.e., the separation between the geoid and the ellipsoid) due to the fact that we can compute orthometric height differences from the GPS ellipsoidal height differences corrected for the geoid undulations. Regional contoured geoid map also has its applications in geophysics, for example, studies related to the subduction zones and tectonic plate boundaries in the South East Asian region can be performed using geoidal information.

A cheap source of geoid undulations and other gravity field related quantities is the Earth Gravity Models (EGMs) defined by a set of potential coefficients. In this study we will use EGMs developed at the Ohio State University, i.e., OSU81 and OSU86 models [*Rapp, 1981 and Rapp et al, 1986*]. These models used satellite data by considering orbital variations, satellite altimeter data and terrestrial gravity data in their determination. The maximum degree of expansion of OSU81 is 180, while OSU86F is 360.

We suppose that the earth's disturbing potential (T) external to a bounding sphere of radius R_B is described by a set of fully normalized potential coefficients:

$$T(\phi, \lambda, r) = \frac{kM}{r} \sum_{n=2}^{\infty} \left[\frac{R_B}{r}\right]^n \sum_{m=0}^{n} [\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin\lambda m]\, \bar{P}_{nm}(\sin\phi) \quad \ldots (1)$$

where

kM	geocentric gravitational constant;
r, ϕ, λ	geocentric coordinates;
$\bar{C}_{nm}, S_{nm}$	fully normalized potential coefficients;
$\bar{P}_{nm}$	fully normalized Legendre function of degree n and order m;
R_B	radius of a sphere completely inside the earth, called the Bjerhammar sphere.

Equation (1) can be used to calculate the geop-spherop separation by using the Brun's equation $N = T/\gamma$, where γ is the normal value of gravity near the given location. If we evaluate (1) at the geoid that is approximated by the ellipsoid we can write for the geoid undulation:

$$N(\phi, \lambda, R_E) = \frac{kM}{R_E \gamma_E} \sum_{n=2}^{\infty} \left[\frac{R_B}{R_E}\right]^n \sum_{m=0}^{n} [\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda]\, \bar{P}_{nm}(\sin\phi) \qquad \dots (2)$$

where R_E is the geocentric radius vector to the ellipsoid, $R_E > R_B$. Although equation (2) indicates a sum to infinity, in practice the sum is to a finite degree such as 20, 36, 180, etc.

The equation of the undulation from (2) yields a point value that is affected by the degree of truncation and the accuracy of the coefficients among other things. Equation (2) with the OSU81 potential coefficients set complete to degree and order 180 has been evaluated for the Malaysian region ($0° \leqslant \phi \leqslant 8°$, $96° \leqslant \lambda \leqslant 120°$) using a slightly modified program described by Rizos (1979). These values are shown in Figure 1 in the form of a contoured map. This map has been constructed from points of 0° using a slightly modified program described by Rizos (1979). These values are shown in Figure 1 in the form of a contoured map. This map has been constructed from points of $0°.25 \times 0°.25$ grid intersections with GRS80 ellipsoid as the reference ellipsoid (a = 6378137 m, f = 1/298.257). The computational time on the IBM 3081 computer required for the determination of geoid undulations of about 3,070 points was only 52 seconds.

It is clear from Figure 1 that the entire Malaysian region is located on one of the steepest geoids in the world, with a gradient of about -0.4, m/10 km approximately in the east-west direction. This is due to the effects of the two most prominent high and low geoid areas, namely, the New Guinea High (undulation of about + 80 m) and the Indian Low (undulation of about — 100 m) located to the east and west of the Malaysian area, respectively.

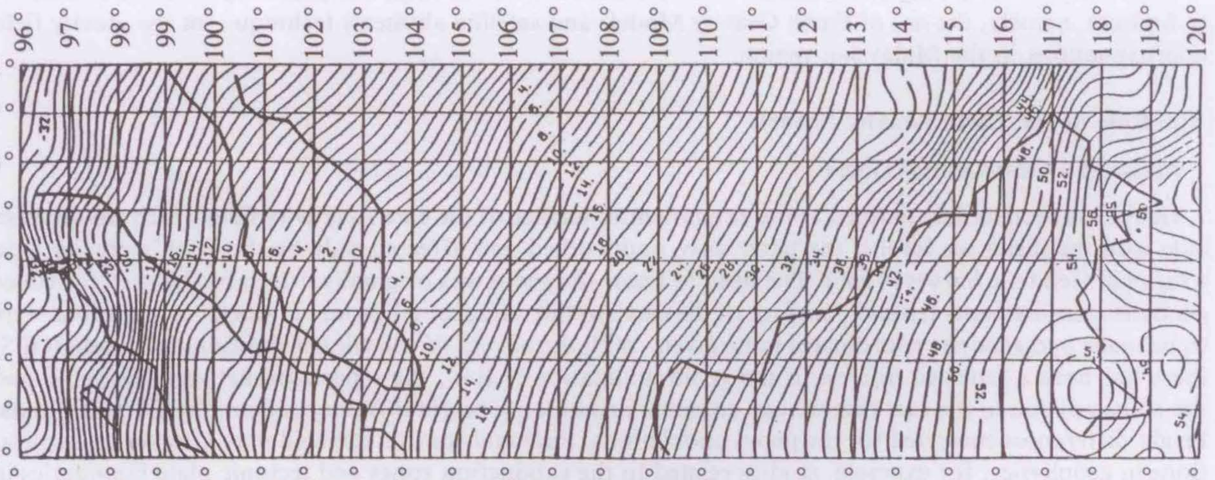

Figure 1 Malaysian Geopotential Geoid from a Spherical Harmonic Expansion to Degree 180 (OSU81) (C.I. = 1 meter, Ref. Ellipsoid = GRS80)

Doppler Undulation Comparisons

In order to check for geodetic accuracy, we next turn to a comparison of the geoid undulations derived from the OSU86E and OSU86F potential coefficients sets with undulations derived from satellite Doppler derived positions. Before any comparison can be made, it is important that the Doppler derived ellipsoidal height be put into the same coordinate system and referred to the same ellipsoid as the geopotential geoid. From Rapp (1983) we can write (ignoring small rotation effects):

$$\Delta h = \cos\phi \cos\lambda \, \Delta X + \cos\phi \, \sin\lambda \, \Delta Y + \sin\phi \, \Delta Z - W\Delta a + \frac{a(1-f)}{W} \sin^2\phi \, \Delta f + (aW + h) \leqslant \Delta L \qquad \ldots (3)$$

where

$\Delta a = a_{new} - a_{old} = 6378137 \text{ m} - 6378145 \text{ m}$

$\Delta f = f_{new} - f_{old} = 1/298.257 - 1/298.25$

$W^2 = 1 - e^2 \sin^2 \phi$

a is the equatorial radius of the reference ellipsoid, f is the flattening of the ellipsod, and e is the eccentricity of the ellipsoid. Since the five available Doppler stations in Peninsular Malaysia are given in the Doppler system (NWL-9D), the following translation and scale parameters were used to convert the Doppler system to a geocentric system [*see Boucher et al., 1985*]:

$\Delta X = -\ 10.6$ cm
$\Delta Y = 69.7$ cm
$\Delta Z = 490.1$ cm
$\Delta L = -\ 0.604$ cm

If h^D is the ellipsoidal height of a station derived through Doppler satellite positioning technique, after conversion to a geocentric system and properly scaled, the Doppler undulation is:

$$N^D = h^D - H \qquad \ldots (4)$$

where H is the orthometric height of the point. The geopotential geoid given by equation (2) was evaluated at the corresponding Doppler stations using the computer program described in Rapp (1982). The comparison of Doppler derived undulations in Peninsular Malaysia with values from OSU86E and OSU86F models are given in Table 1 and Table 2, respectively. The results shown in these tables indicate that geopotential geoid from OSU86E or F with expansion complete to degree and order 250 gives the best root-mean-square fit (rms = 0.53) with respect to the Doppler derived geoid undulations. The largest difference occurs at the Kertau Doppler station ($\phi = 3°.4638$, $\lambda = 102°.6217$, H = 267.9 m). If this station is omitted, the $N^D - N^{250}$ differences using OSU86E and OSU86F given rms values of 0.36 and 0.45, respectively.

Table 1 Comparison of Doppler Derived Undulations in Peninsular Malaysia with Values from OSU86E Model

DOPPLER STATION			OSU86E			$N^D - N^{180}$ (m)	$N^D - N^{250}$ (m)	$N^D - N^{360}$ (m)
Lat.	Long.	N^D (m)	N^{180} (m)	N^{250} (m)	N^{360} (m)			
6°.1397	100°.3849	−11.57	−11.31	−11.35	−11.25	−0.26	−0.22	−0.32
6°.0387	102°.3205	−5.40	−5.24	−5.34	−5.31	−0.16	−0.06	−0.09
3°.4638	102°.6217	1.50	0.30	0.54	0.43	1.20	0.96	1.07
3°.0247	101°.4456	−2.52	−2.21	−2.17	−2.20	−0.31	−0.35	−0.32
1°.3765	103°.6080	7.19	6.39	6.60	6.57	0.80	0.59	0.62
					RMS	0.67	0.54	0.59

Marine Gravity Field from Satellite Altimeter Data

Background

The satellite born radar altimeter represents one of the major successes in satellite geodesy. The altimeter data has been acquired over all the ocean areas between the latitude limits of ±72°. Figure 2 illustrates the geometry of geoidal and sea surface height determination by satellite altimeter. The satellite serves as a stable platform from which a radar altimeter can measure the distance to the instantaneous ocean sur-

Table 2. Comparison of Doppler Derived Undulations in Peninsular Malaysia with Values from OSU86F

DOPPLER STATION			OSU86F			$N^D - N^{180}$ (m)	$N^D - N^{250}$ (m)	$N^D - N^{360}$ (m)
Lat.	Long.	N^D (m)	N^{180} (m)	N^{250} (m)	N^{360} (m)			
6°.1397	100°.3849	—11.57	—11.03	—11.06	—10.97	—0.54	—0.51	—0.60
6°.0387	102°.3205	—5.40	—5.01	—5.10	—5.06	—0.39	—0.30	—0.34
3°.4638	102°.6217	1.50	0.49	0.74	0.63	1.01	0.76	0.87
3°.0247	101°.4456	—2.52	—2.02	—2.00	—2.01	—0.50	—0.52	—0.51
1°.3765	103°.6080	7.19	6.52	6.74	6.71	0.67	0.45	0.48
					RMS	0.66	0.53	0.59

face. Knowing the orbital altitude of the spacecraft above the reference ellipsoid, the geoidal height can be obtained as (seen Figure 2):

$$N = h - \zeta = (r - a) - \zeta \qquad \ldots (5)$$

where

N geoidal height;
r geodetic height of the spacecraft;
a measured altitude of spacecraft above the ocean surface, corrected for a number of instrumental and geophysical effects;
h height of sea-surface above the reference ellipsoid; and
ζ sea-surface topography

The sea-surface topography (SST) mentioned above is the departure of the mean sea level from the geoid. The causes of SST include ocean currents, water density variations, as well as air pressure and wind stress. Since SST is known to be less than 1 m in amplitude, the mean sea-surface heights obtained from satellite altimetry will closely represent the geoid to an accurancy of approximately ± 1 m.

Until recently, the main sources of the altimeter data have been the Geos-3 and Seasat altimeter satellites. The Geos-3 data was acquired in the time period 1975—1978, while the Seasat data was limited to the time period June through October 1978. The results to be presented in this study will be based on a combined Geos-3/Seasat database developed through a series of studies conducted at the Ohio State University. A brief review of these studies can be found in Majid (1987b).

Method of Gravity Field Recovery

We have previously mentioned that the primary information from the Geos-3/Seasat missions is a set of time averaged sea-surface heights. Ignoring the effects of the sea-surface topography, we can use the altimeter-implied geoid undulations (h) to compute other gravity field related quantities such as gravity anomalies ($\triangle g$) and components of the deflections of the vertical (ξ , n). The technique that will be used for the gravity anomally recovery and sea-surface height estimation is least squares collocation as described by Moritz (1980). The prediction of the signal and its accuracy estimate is carried out with the following equations:

$$\hat{s} = C_{sh}(C_{hh} + D)^{-1}\,(\underline{h} - \underline{h}_R) + s_R \qquad \ldots (6)$$

$$m^2_{\hat{s}} = C_{ss} - C_{sh}\,(C_{hh} + D)^{-1}\,C_{hs} \qquad \ldots (7)$$

where

$\underline{h}$ column vector of the altimeter-implied undulation;
C_{sh} row vector containing covariances (referred to the reference field) between the quantity being predicted and the given geoid undulations;
C_{hh} square, symmetric matrix containing the covariances (referred to the reference field) between

the geoid undulations;

D error covariance matrix of the observed geoid undulations which was taken to be a diagonal matrix whose elements corresponded to the square of the standard deviation of the altimeter measurement;

C_{ss} expected mean square value (referred to the reference field) of the quantity being predicted;

s_R , h_R signal being predicted and geoid undulation implied by the reference field.

The reference gravity field model used in the computations was the OSU81 potential coefficients set complete to degree and order 180. These coefficients were used to calculate values of h_R and $\triangle g_R$ on a 0°.25 grid from which interpolations were made to a specific altimeter observation or a prediction grid point.

The auto- and cross-covariance functions involving geoid undulation and gravity anomaly with respect to a reference field complete to degree 180 were evaluated using the subroutine COVA described in Tscherning et al. (1974). The actual covariance function used is the total covariance function which can be described as follows [for details, see *Majid, 1987b*]:

$$K(P,Q) = K_R\,(P,Q) + \triangle K\,(P,Q) \qquad \ldots(8)$$

where

$K\,(P,Q)$ covariance function of the disturbing potential;
$K_R\,(P,Q)$ N^{th} order covariance function;
$\Delta K\,(P,Q)$ error covariance function implied by the potential coefficient noise model.

The auto- and cross-covariances of other gravimetric quantities (i.e., C_{hh}, $C_{\triangle g \triangle g}$, $C_{\triangle gh}$, etc). can be derived from K(P,Q) according to the law of propagation of covariance and taking into account the well known functional relationships between these quantities with the disturbing potential. In order to save computer time, the covariances are set up in a table at an interval of 0°.05 prior to the actual computations. In the present application, the covariance function will be tailored to a specific area so that it will yield the observed residual undulation variance. With this procedure, an approximate local representation of the covariance functions can be achieved.

Test Computations and Results

The 1°x1° test area is located off the east coast of Johor, in the Pulau Tioman area (Figure 3). It's geographic region is defined by $2° \leqslant \phi \leqslant 3°$ and $104° \leqslant \lambda \leqslant 105°$. The prediction run was made based on the following prediction choice.

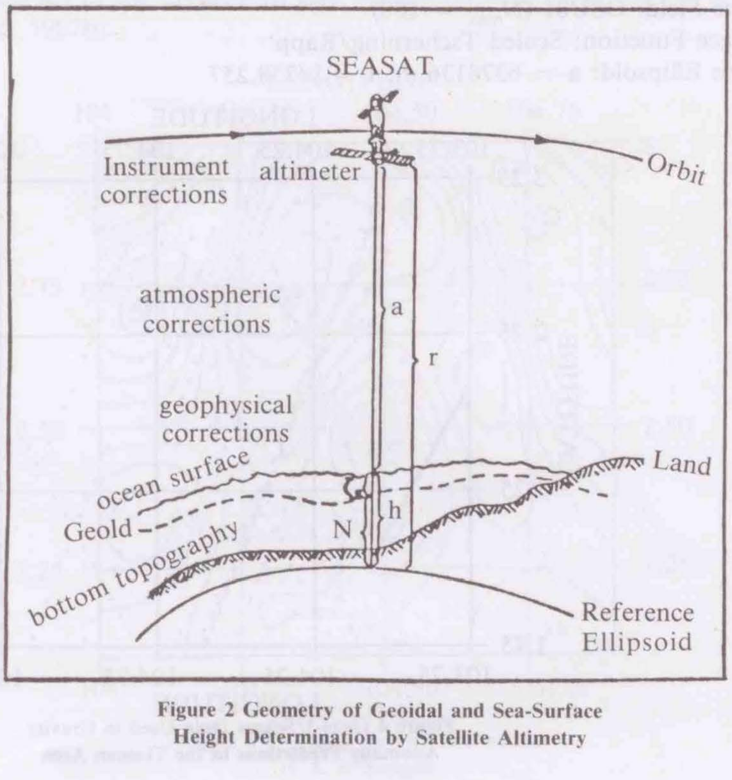

Figure 2 Geometry of Geoidal and Sea-Surface Height Determination by Satellite Altimetry

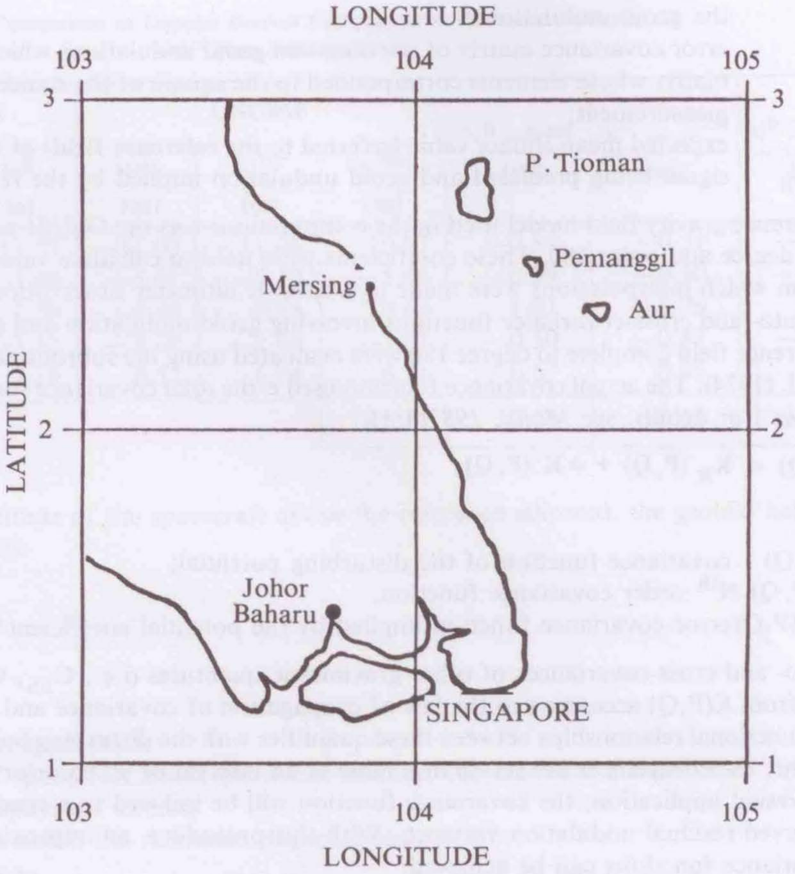

Figure 3 Location of the Test Area

Number of Prediction Sets: 1
Blocksize: 1° x 1°
Data Border: 0°.25
Data Number: 300 points (for one matrix inversion)
Grid Interval: 0°.1
Reference Field: OSU81 ($N_{max} = 180$)
Covariance Function: Scaled Tscherning/Rapp
Reference Ellipsoid: a = 6378136 m, f = 1/258.257

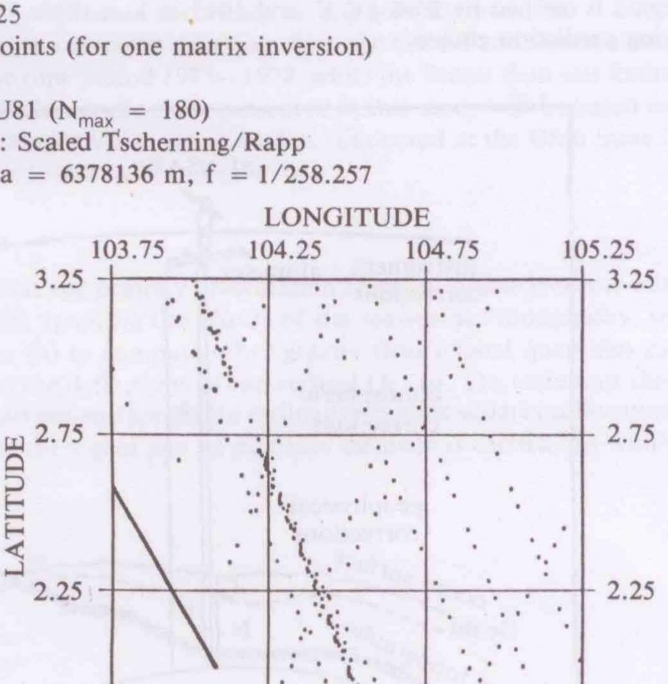

Figure 4 Geos-3/Seasat Data Used in Gravity Anomally Predictions in the Tioman Area

The Geos-3/Seasat data used in this prediction run is shown in Figure 4. The estimated sea-surface heights and the predicted gravity anomalies are shown in Figures 5 and 6, respectively. As can be seen in Figure 6, the high positive anomalies (about 50 mgals) reflect the signatures of the islands in this area, namely, Tioman, Pemanggil and Aur islands. On the other hand, the low anomalies (about 5 mgals) to the east of Tioman island may be associated with the bathymetry and/or geology of the oceanic crust.

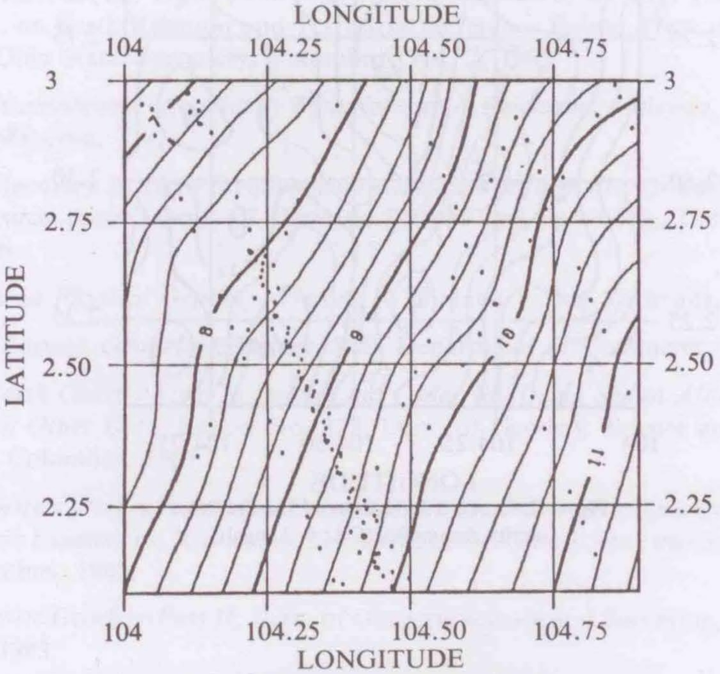

Figure 5 Estimated Sea-Surface Heights
(C.I. = 0.25 meter)

In Figure 7 we present the plot of the estimated standard deviations of the predicated gravity anomalies ($m_{\Delta g}$). The accuracy estimates (computed using equation (7)) show $m_{\Delta g}$–values of about ± 9 mgals in areas with good altimeter data coverage, but deteriorated to ± 16 mgals where no altimeter tracks are available (see Figure 7). The accuracy of the predictions also depend on data noise and the scale of magnitude of the covariances (Majid, 1987b).

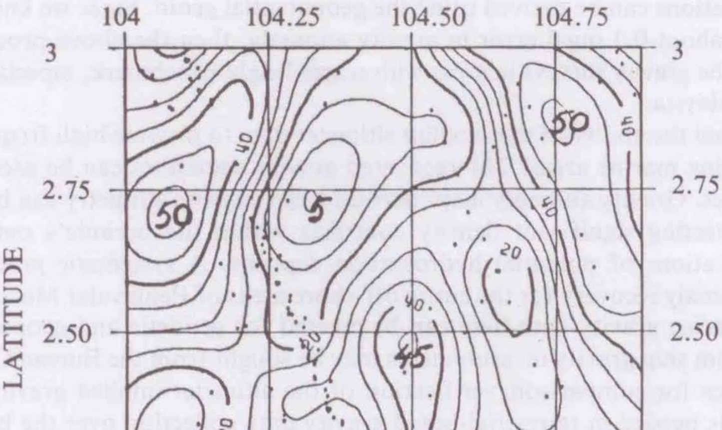

Figure 6 Predicted Gravity Anomalies
(C.I. = 5 mgal)

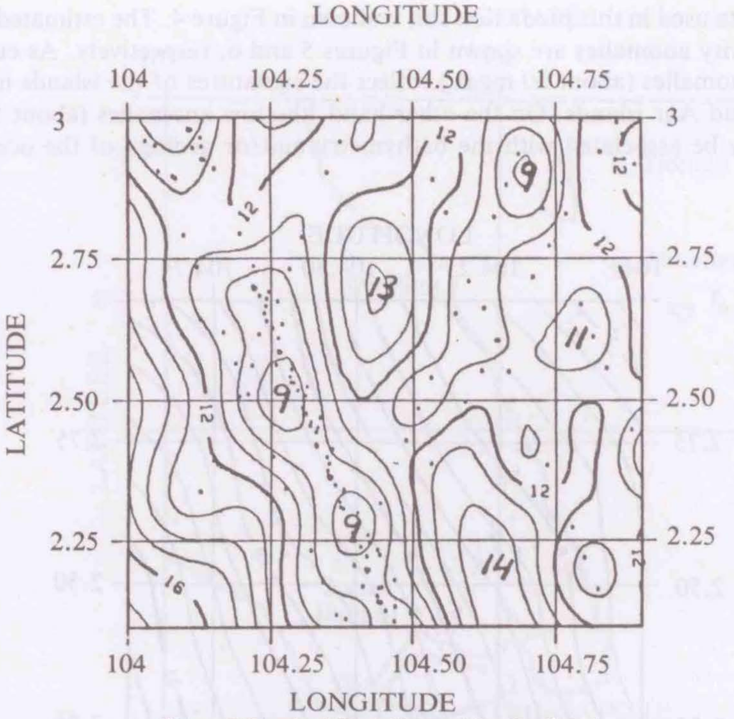

Figure 7 Standard Deviations of the Predicted Gravity Anomalies (C.I. = 1 mgal)

Conclusions

From the analysis made so far it is understood that the Earth Gravity Models (EGMs) and satellite altimeter data can significantly contribute in the gravity field approximations in the Malaysian area. EGMs defined by a set of high degree potential coefficients can be effectively used to generate the long and medium wavelength gravity field information (resolution to 100 km). The comparison with the Doppler geoid undulations indicates that the geopotential geoid (OSU86E, $N_{max} = 250$) can be used in conjunction with satellite derived ellipsoidal height to compute the orthometric height accurate to better than ± 1 meter. It is anticipated that if gravity measurements are carried out where the gravity station positioning and elevation in the survey will be by Global Positioning System (GPS) operating in differential mode, the orthometric heights of the gravity stations can be derived using the geopotential geoid. Since we know that a one meter error in height implies about 0.3 mgal error in gravity anomaly, then the above procedure may provide a practical solution for the gravity surveys in areas with scarce height benchmark, especially in the hinterland areas of Peninsular Malaysia.

We also have illustrated the ability of the satellite altimeter data to provide high frequency gravity information in the surrounding marine areas. The recovered gravity anomalies can be used for both geodetic and geophysical purposes. Gravity anomaly maps derived from satellite altimetry can be used to scan large off-shore areas for detecting significant density contrasts within the oceanic's outer crust, and thus providing indirect indications of potential hydrocarbon deposits. A systematic production run will be required for gravity anomaly recovery for the entire off-shore areas of Peninsular Malaysia. Consequently, an altimeter-implied marine gravity data-base can be created for geodetic and geophysical applications. Existing gravity data from ship gravity measurements may be sought from the Bureau Gravimetrique International (BGI) in France for comparison/verification of the altimeter-implied gravity anomalies.

A continuous effort is needed in terrestrial-based gravity data collection over the land areas of Peninsular Malaysia. A sufficiently dense (at least 300 points in a 1° × 1° area) and uniformly distributed gravity data can be used together with digital terrain model for precise geoid determination (accuracy of better than 10 cm). For this purpose, we suggest that a test-area with the above data density and distribution be created for geoid studies. The method of least squares collocation may be utilized for the computations. In order to verify the computed gravimetric-geoid, and independent geoid can be derived from a GPS network, which is established in the test area with it's stations located at the known height benchmarks.

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