A STUDY OF GEOPOTENTIAL GEOID IN THE PENINSULAR OF MALAYSIA

Shahrum Ses
Fakulti Ukur
Universiti Teknologi Malaysia

Synopsis

Geoidal heights can be computed for a single point value or a grid of values. A program to compute geoidal heights from a set of high degree potential coefficients was developed. Backward recurrance formulas have been used to evaluate the value of normalized Legendre functions. The regional and global geopotential geoid evaluated from the available sets of potential coefficients were shown in the form of contoured maps. The computed geoidal heights derived from different sets of potential coefficients were compared with Doppler derived values at six points in Peninsular Malaysia. The results indicate that of the models tested OSU86 gives the best solution to the geopotential geoid in the region.

Introduction

The geoid has been loosely defined as the equipotential surface of the earth's gravity field which would coincide with the mean sea level if the latter were undisturbed and affected only by the earth's gravity field. It is an important surface to which many geodetic observations are related. While the geodetic coordinates of the point are referred to the ellipsoid, orthometric heights are referred to the geoid. The relationship between the terrain, geoid, and ellipsoid is shown in Figure 1. The geoid is of increasing importance in modern development of geodesy, as it needs to be known in order to convert ellipsoidal heights to orthometric heights.

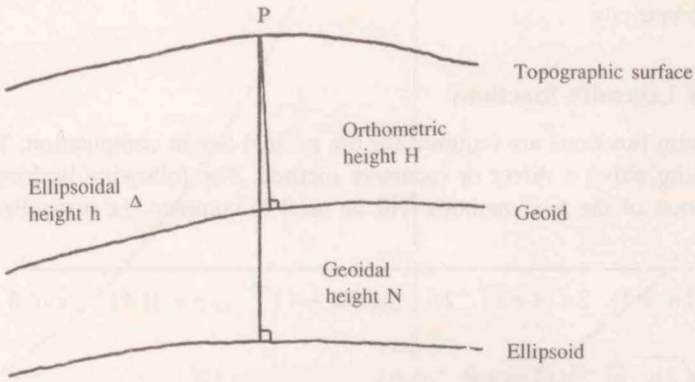


Figure 1: Topographic surface, geoid and ellipsoid

If the terrain point P is to have all three defining parameters referred to the ellipsoid, a knowledge of the geoidal height is required.

Geoidal heights may be computed if a global estimate of the gravity anomaly field is used in the Stokes' equation. However, if we are given a set of potential coefficients describing the gravitational potential of the earth, the geoidal heights may also be computed. These coefficients are determined from a combination of a satellite orbit perturbations, satellite altimeter data, and mean terrestrial gravity data. Once determined, they are valid everywhere on the earth's surface, i.e. thay can be used to compute geoidal height for any given latitude and longitude. Examples of such coefficients are the GEM10B model (Lerch et al [1981]), the OSU81 and OSU86 models (Rapp [1981] and Rapp et al [1986]) and the GPM2 model (Wanzel [1985]).

Approach used to compute geoidal heights

The earth's disturbing potential (T) is given by a set of fully normalized potential coefficients;

$$T(\theta, \lambda, r) = GM/r \sum_{n=2}^{\infty} [a/r]^{n} \sum_{m=0}^{n} \left[\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda\right] \overline{P}_{nm} (\cos\theta) \quad \text{... (1)}$$

where

where

GM	geocentric gravitational constant
r, θ, λ	geocentric coordinates
$\overline{C}_{nm}, \overline{S}_{nm}$	fully normalized potential coefficients
$\overline{P}_{nm}$	fully normalized Legendre function of degree n and order m
a	equatorial radius of a reference ellipsoid.

The above equation can be used to calculate the geoidal height $N(\theta, \lambda, r)$ by using the Brun's formula $N=T/\gamma$, where γ is the normal value of gravity at the given point:,

$$N(\theta, \lambda, r) = GM/r\,\gamma \sum_{n=2}^{\infty} [a/r]^n \sum_{m=0}^{n} \left[\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda\right] \overline{P}_{nm}(\cos\theta) \quad \text{... (2)}$$

The lower even degree zonal coefficients i.e. $\overline{C}_2, \overline{C}_4$ and $\overline{C}_6$ have to be corrected to remove the effect of the normal gravity field. The correction can be computed using the series expansion of the normal gravity fiels (Heiskanen and Morits [1967];

$$\Delta C_{2n} = (4n+1)^{-\frac{1}{2}} (-1)^0\, 3e^{2n} (2n+1)^{-1} (2n+3)^{-1} \left[1 - n + (5n\,\overline{C}_2/e^2)\right] \quad \text{... (3)}$$

where

ΔC_{2n}	fully normalized correction term
e	first eccentricity

Method to compute the Legendre functions

The normalized Legendre functions are required for the geoidal height computation. These normalized values may be computed by using either a direct or recursive method. The following backward recurrance formulas derived from a combination of the two methods will be used to compute the normalized Legendrs functions;

$$\overline{P}_{nm}(t) = \sqrt{2(2n+1)\ 2n(4n)^{-1}\ 2n-1(4n-4)^{-1} \ldots n+1(4)^{-1}}\ \sin^n\theta$$

$$\overline{P}_{nn-1}(t) = \sqrt{2n}\ \overline{p}_{nm}(t) \cot\theta$$

$$\overline{p}_{nm}(t) = 2(m+1)\cot\theta \sqrt{(n-m)^{-1}(n+m+1)^{-1}}\, \overline{P}_{nm+1}(t)$$

$$- \sqrt{(n-m-1)(n+m+2)}\,(n-m)^{-1}(n+m+1)^{-1}\, \overline{P}_{nm+2}(t) \quad \text{... (4)}$$

where

t	$\cos\theta$
θ	colatitude of computation point

Description of the Program

The program is written to be interactive and prompts the user throughout. The program offers the following options;

A. Choice of case

Geoidal heights can be computed for two differents cases. These include a single point value or a grid of values. The data required is depending upon the case selected by the user.

B. Maximum degree (N max) required

Although equation (2) indicates a sum to infinity, in practice the sum is to a finite degree such as 36,180,360, etc. The user may specify any value for N max, up to a maximum of 360, for the computation of geoidal height.

C. Normal gravity field

The parameters defining the geocentric reference system used in this program $(a, f, \bar{C}_2 \text{ and } GM)$ could be changed if desired. By changing f or $\bar{C}_2$ the coefficients of the normal gravity field (equation (3)) are altered.

Discussion on the method of computation

In order to check the stability of computing the normalized Legendre functions using equations (4), the related subroutine was tested for different latitudes and varying degree and order. The normalized values obtained from different values of N max were compared. For each case the normalized Legendre functions agreed. This shows the stability of the method being used to compute thhese values.

Next, the subroutine was timed for the calculation of the normalized values for varying degree and order. The following graph in Figure 2 shows the C.P.U. times required to compute the normalized values for different N max.

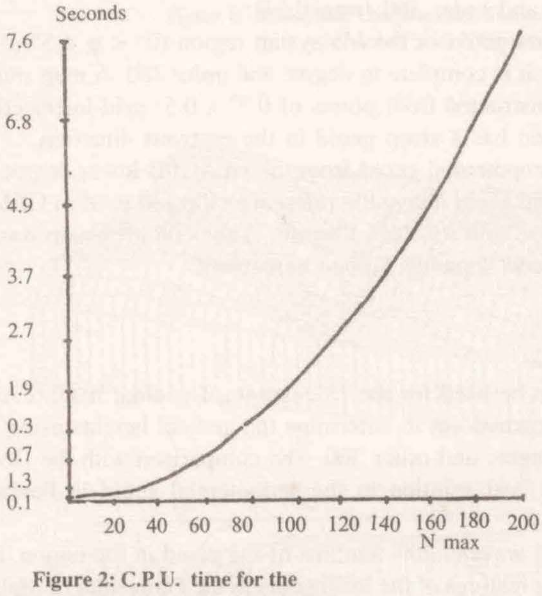


Figure 2: C.P.U. time for the normalized Legendre functions calculation.

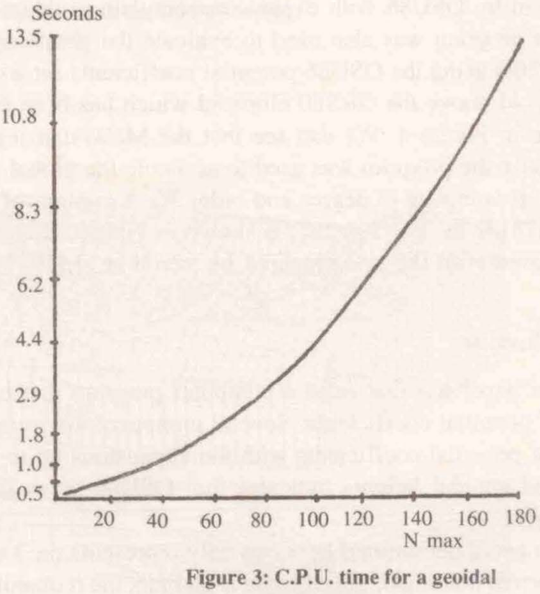


Figure 3: C.P.U. time for a geoidal height calculation

The main program was also timed for a single geoidal height calculation and the second graph in Figure 3 shows the results for varying degree and order. These times include the computation of the normalized Legendre functions.

The accuracy of th ecomputed geoidal heights is mainly dependent upon two fctors. First, the accuracy of the potential coefficients being used and second, the degree and order at which the infinite series in equation (2) is truncated.

Discussion of the results

Geoidal heights at six Doppler points in Peninsular Malaysia were computed from the different sets of potential coefficients. The results were then compared with the geoidal heights derived from satellite Doppler derived positions. Prior to this, the Doppler derived cartesian coordinates which are given in WGS72 have been transformed to GRS80 using the parameters given by Cross. [1987].

Table 1: Comparison of Doppler derived geoidal heights in Peninsular Malaysia with values from different sets of potential coefficients

Doppler points		Geoidal heights (m)				1-2	1-3	1-4
Lat	Long	1 Dopp	2 Gem10B 36	3 OSU81 180	4 OSU86 360			
3.4638	102.6217	0.80	3.31	-0.12	0.25	-2.51	0.92	0.41
3.0247	101.1156	-3.22	-3.31	-3.17	-2.25	-3.53	-0.05	-0.98
6.0387	102.3205	-6.12	-3.09	-6.19	-5.39	-3.03	0.07	-0.73
1.3765	103.6080	6.50	8.21	7.99	6.46	-7.71	-1.49	0.04
6.1397	100.3849	-12.28	-8.70	-12.82	-11.26	-3.58	0.54	-1.02
1.4689	103.2564	4.57	7.13	6.22	5.00	-2.56	-1.65	-0.43
					RMS	0.65	0.96	0.53

The results shown in Table 1 indicate that the best solution considering the root mean square of the difference is given by OSU86 with expansion complete to degree and order 360 (rms=0.53).

The program was also used to evaluate the geopotential geoid in the Malaysian region ($0° < \phi < 8°$, $96° < \lambda < 120°$) using the OSU86 potential coefficients set which is complete to degree and order 360. A map showing the geoid above the GRS80 ellipsoid which has been constructed from points of 0.5° x 0.5° grid intersection is shown in Figure 4. We can see that the Malaysian region has a steep geoid in the eastwest direction.

Finally the program was used to compute the global geopotential geoid from the GEM10B lower degree field which is complete to degree and order 36. A contour of the geoid above the reference ellipsoid used in GEM10B (a=6378138 m, f=1/298.257) is shown in Figure 5 together with its block diagram. The contoured map was then compared with the one prepared by Lerch et al [1981] and showing a good agreement.

Conclusions

This paper has discussed a computer program that can be used for the calculation of geoidal heights from a set of potential coefficients. Several computations were carried out to determine the geoidal heights using three sets of potential coefficients with the expansions up to degree and order 360. The comparison with the Doppler derived geoidal heights indicates that OSU86 gives the best solution to the geopotential geoid in Peninsular Malaysia.

The geoid determined here can only represents the long wavelengths features of the geoid in the region. Since the current holding of gravity data is lacking, the remaining features of the local geoid in the Peninsular of Malaysia cannot be evaluated. In future, the long wavelength geoidal heights information could be combined with the remaining features evaluated gravimetrically to determine a local gravimetric geoid in the region.

Acknowledgements

I wish to acknowledge the guidance given by Prof. Dr. Paul A. Cross at the Department of Surveying, University of Newcastle upon Tyne.

The Doppler data used in this study is provided by The Department of Surveying and Mapping, Malaysia.

References

Cross, P.A., Private Communication, Dept. of Surveying, University of Newcastle upon Tyne, 1987.

Heiskanen, W.A. and H. Moritz, *Physical Geodesy*, W.H. Freeaman and Company, San Francisco and London, 1967.

Lerch, F.J., F. Putney, S. Klosko and C. Wagner, *Earth Models for Oceanographic Applications (GEM10B and !0C)*, Marine Geodsy, Vol.5, 145-187, 1981.

Rapp, R.H., *The Earth Gravity Field to Degree and Order 180 Using SEASAT Altimeter Data, Terrestrial Gravity Data, and Other Data*, Report No.322, Dept. of Geodetic Science and Surveying, Thhe Ohio State University, Columbus, 1981.

Rapp, R.H. and Cruz, J.Y., *Spherical Harmonic Expansions of the earth's Gravitational Potential to Degree 360 Using 30' Mean Anomalies*, Report No. 376, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, 1986.

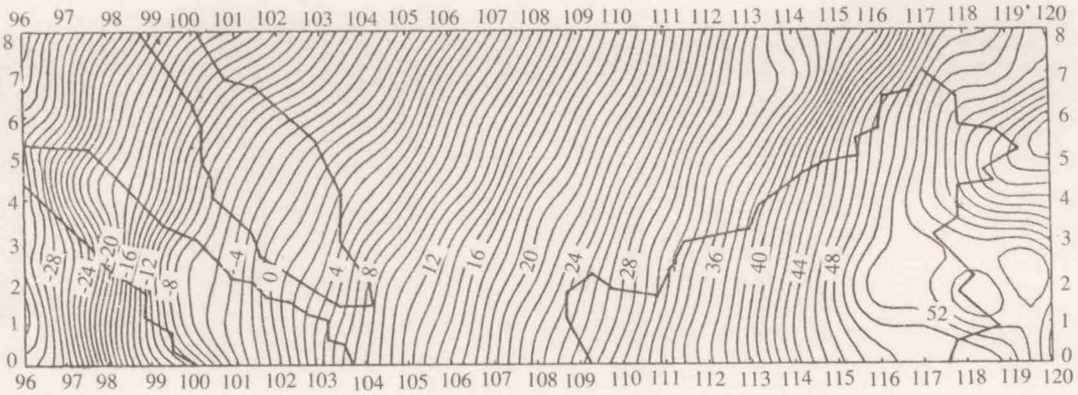
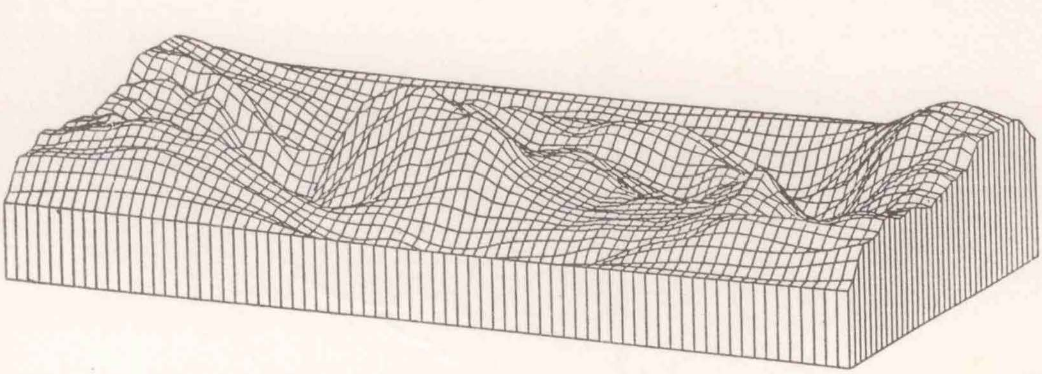


Figure 4: Malaysian Geopotential Geoid computed from the OSU86 Model (C.I. = 1 metre, Ref. Ellipsoid = GRS80)



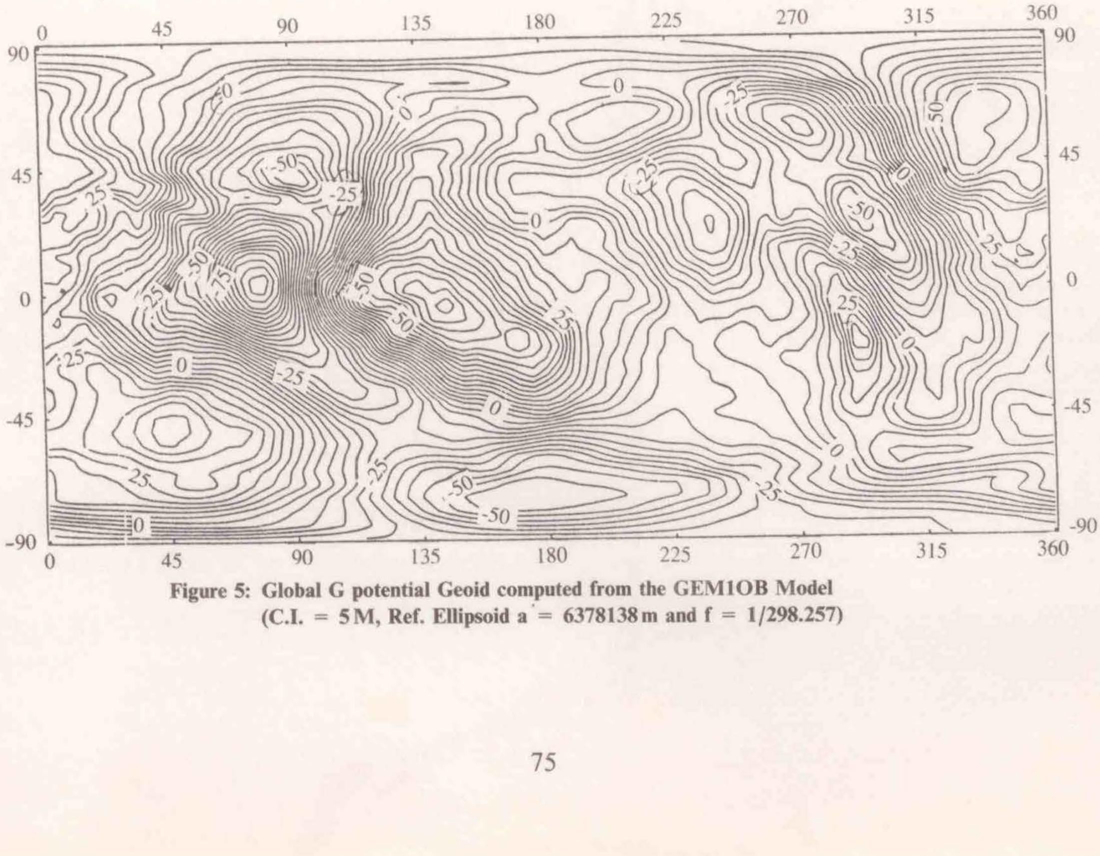


Figure 5: Global G potential Geoid computed from the GEM1OB Model (C.I. = 5 M, Ref. Ellipsoid a = 6378138 m and f = 1/298.257)