DYNAMIC SECURE ECONOMIC DISPATCH FOR LARGE SCALE POWER SYSTEMS

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Synopsis

Dynamic multi-period economic dispatch for large-scale power systems is considered. The formulation presented caters for loading and deloading rates, limits on generators outputs, spinning reserves requirements and group power import-export limits. It is also suitable for implementation within a constraint relaxation strategy. The solution algorithm is based on a Dantzig-Wolfe decomposition, which yields a capacitated transshipment subproblem along with a master problem solved by the Revised Simplex method. The computational efficiency of the algorithm renders it suitable for on-line dispatch.

Indexing terms: Power system control, Load dispatching, Modelling, Mathematical programming.

List of principal symbols

G_{ij}	instantaneous output of generator i at the end of period j
ΔG_{ij}	change in output of generator i during periof j
G_i^M	maximum stable output level of generator i
G_i^m	minimum stable output level of generator i
T_j	duration of period j
D_j	total demand (load) at the end of period j
bG_i^+	incremental cost of generator i
δG_i^-	maximum loading rate of generator i
δ	maximum deloading rate of generator i
S_{ij}	spinning reserve contribution of generator i at period j
SL_i	spinning reserve level of generator i
ST_j	total spinning reserve requirement during period j
p	number of periods
I_g	import constraint limit for group g
E_g	Export constraint limit for group g

Introduction

Economic load dispatch is essential for real-time control of power system operation. It is the process of allocating the total generation required among the available thermal generating units so that the cost of energy is minimised subject to physical and operational constraints, assuming that a hydro-generation and a thermal unit commitment have been previously determined. This important problem has received a great deal of attention during the past two decades, manifested in a large and growing body of literature. The overwhelming majority of reported work however, deals with static economic dispatch; that is the horizon is divided into periods and the dispatch is optimised period by period.

The problem is frequently formulated by assuming instantaneous loading and deloading characteristics for the thermal units. In practice, however, there are maximum loading and deloading rates for most units when operating between their stable minimum and maximum generation limits. This means that variabIęs representing generation at different instants of time are related to each other through the loading and deloading rates. Thus the dispatch problem is, in fact, dynamic.

Solution by dividing the scheduling horizon into period and minimising the cost of generation period by period enforcing the rate constraints from one period to the next recognises the dynamic nature of the problem, but this

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approach can lead to suboptimal solutions. To illustrate this consider a simple example put forward in [5], comprising three on-line generating units; each running at or above its minimum generation level G_i^m. The scheduling horizon is divided into five periods. The maximum stable generation level of each generator is 600, and the loading rate is 150 per period. The incremental costs of the generators are 0.47, 0.57, 0.67 and the initial generation of each is 500, 0, 0 respectively. The demand, discounted by the sum of the minimum generation levels of the units, is initially 500 and increases by 200 in each interval to reach 1500 at the end of the fifth. Table 1 presents the schedule resulting from solving the problem period by period sequentially while enforcing the loading rates constraints. The cost of the schedule is seen to be more than that of the optimal schedule presented in table 2.

Table 1 Period by period dispatch

Period / Generator	1	2	3	4	5	Cost
1	600	600	600	600	600	1410.00
2	100	250	400	550	600	1083.00
3	0	50	100	150	300	402.00
					Total	2895.00

Table 2 Optimal dispatch

Period / Generator	1	2	3	4	5	Cost
1	550	600	600	600	600	1386.50
2	150	300	450	600	1168.50	
3	0	0	50	150	300	335.00
					Total	2890.00

However, the need for dynamic multi-period solution of the economic dispatch problem does not stem solely from considerations of optimality, but also, and more importantly, from considerations of operational feasibility. As rightly observed in [14], one of the recurring problems facing dispatchers every day is how to operate the system during periods of high load pickup, such that there is sufficient generation to follow, while still maintaining reasonable reserve margins. Static dispatch leads to the least expensive units being run close to their limits during the early stages of the load pickup, leaving the more expensive units for the final stages. But this may lead to inhability to meet the load pickup during the latter while maintaining enough reserve.

Thus consideration of operational feasibility and optimality argue for adopting a dynamic multi-period formulation of the dispatch problem, incorporating spinning reserve constraints. This would allow dispatch to be carried out on a rolling horizon basis with the dispatch problem formulated and solved continually over, say five or six look-ahead periods using the most up to date estimates of demand.

Study of dynamic economic dispatch appears to have been started by Bechert and Kwanty [1, 8]. They applied optimal control theory to develop and synthesise the optimal feedback controller. But their work was limited to a two-generator system due to computational problems. Patton [10] used quadratic programming to solve for optimal generator output trajectories, and tested the method on a four-generator system. Bechert and Chen [2] proposed a multi-pass dynamic programming approach and obtained optimal generator output trajectories for up to five generators.

All the above methods suffered severe dimensionality limitations. Ross and Kim [12] proposed a computationally more efficient dynamic programming successive approximations algorithm, and reported solving a problem involving 15 generating units and 16 dispacth periods. However, their formulation did not cater for spinning reserve constraints, nor for group import-export limits Wood [14] included spinning reserve constraints in his model, but proposed a feasible suboptimal solution. Irving and Sterling [6] proposed a model incorporating loading/deloading constraints, spinning-reserve constraints and group import-export limits. They showed that the resulting large linear programming model can be solved efficiently employing the dual revised simplex algorithm and reported that a problem involving 100 generating units and 5 dispatch periods was solved with modest computational resources.

The work reported in this paper is similar in intention to that of Irving and Sterling [6]. However, different modelling strategy and solution techniques are employed to achieve a degree of computational efficiency suitable for online economic dispatch. In this strategy, use is made of the Dantzig-Wolfe decomposition principle. This principle has been applied to economic dispatch [11, 13], but within a single-period, static framework.

Problem Formulation

Basic Model

It is customary to divide the dispatch horizon into a number of periods and build a model to satisfy the generation-load balance at the end of each period [6]. The resulting oprimising problem, taking into account loading and deloading rate constraints, would be a large linear program.

In contrast, it is assumed, in the present work, that during each period, the load varies linearly. The resulting trapezoidal approximation of the load curve is more accurate than the usual step approximation, particularly when the lengths of period are varied. Moreover, the resulting basic model is a capacitated transshipment problem, which is amenable to much more efficient solution than the corresponding standard linear program.

Let index i denote generators and indices j and l denote periods, with l = j – 1.

Assuming that the generation level of each generator varies linearly during each period, the cost of energy produced by generator i during period j can be written as

$$z_{ij} = b_i \bar{G}_{ij} T_j$$

where $\bar{G}_{ij} = 0.5\ (G_{ij} + G_{il})$.

Thus the overall objectives function is:

$$\min \sum_i \sum_j \alpha_{ij} G_{ij} \qquad \ldots (2.1)$$

where $\alpha_{ij} = 0.5 b_i (T_j + T_l)\ \forall_j \neq p$ and $\alpha_{ij} = 0.5 b_i T_j$ for $j = p$.

If a generator is shut down during a period, the corresponding α_{ij} is set to very high value to force G_{ij} to remain zero. ... (2.2)

Assuming a point network model, the load requirments can be satisfied by ensuring that the change in total generation during each interval equals the change in load;

$$\sum_i \Delta G_{ij} = \Delta D_j \quad \forall_j \qquad \ldots (2.2)$$

where

$$\Delta G_{ij} = G_{ij} - G_{il} \quad \forall_{i,j} \qquad \ldots (2.3)$$

$$\Delta D_j = D_j - D_l \quad \forall_j \qquad \ldots (2.4)$$

The variable ΔG_{ij} can assume a negative or a positive value depending on whether the generator output is increasing or decreasing. However, to satisfy the implicit non-negativity requirement of linear programming, ΔG_{ij} can be expressed as the difference between two positive variables: ΔG^+_{ij} and ΔG^-_{ij}

This results in the following;

$$\sum_i (\Delta G^+_{ij} - \Delta G^-_{ij}) = \Delta D_j \quad \forall_j \qquad \ldots (2.5)$$

$$\Delta G^+_{ij} - \Delta G^-_{ij} = G_{ij} - G_{il} \quad \forall_{i,j} \qquad \ldots (2.6)$$

The upper and lower limits on generator stable outputs can be expressed as:

$$G^m_i \leq G_{ij} \leq G^M_i \quad \forall_{i,j} \qquad \ldots (2.7)$$

The loading and deloading rates of a generator should be less than or equal to a certain maximum. This constraint can be stated as:

$$\Delta G^+_{ij} \leq \delta G^+_i T_j \quad \forall_{i,j} \qquad \ldots (2.8)$$

$$\Delta G^-_{ij} \leq \delta G^-_i T_j \quad \forall_{i,j} \qquad \ldots (2.9)$$

Treating lower limits in (2.7) by applying the standard device of variable substitution throughout, the objective function (2.1) and constraints (2.5), (2.6), (2.7) and (2.9) constitute a capacitated transshipment problem. Such problems can be solved by a network flow algorithm characterised by simplicity, minimum rounding-off errors and low storage requirements [4]. This algorithm is also fast; it is claimed That it can be over a 100 times faster than solution by a general linear programming code [7].

Additional Constraints

Apart from hardware considerations, the reliability of generation is mainly determined by the ability of the power system to anticipate sudden, unexpected changes in the demand and/or generator outages. This ability is realised by the spinning reserve, the amount of already committed but not yet used generation capacity. The amount of the system total spinning-reserve requirement is usually calculated on the basis of realiability evaluations. As a lower bound, it usually is at least equal to the largest infeed. Besides the determination of the size of the spinning-reserve, its distribution among the running generators is very important and determines both generation costs and the reliability.

The spinning-reserve contribution of a steam thermal unit can be modelled as shown in Figure 1. If the unit is working in the lower region, then its reserve contribution is;

$$S_{ij} = k_i G_{ij},$$

otherwise, it is;

$$S_{ij} = G_i^M - G_{ij}$$

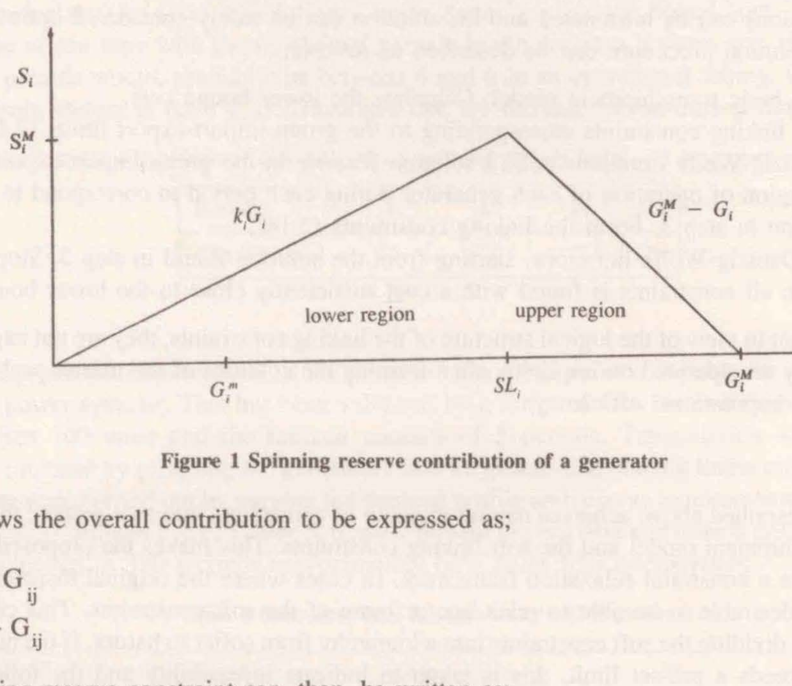

Figure 1 Spinning reserve contribution of a generator

Convexity allows the overall contribution to be expressed as;

$$S_{ij} \le k_i G_{ij} \qquad \text{... (2.10)}$$

$$S_{ij} \le G_i^M - G_{ij} \qquad \text{... (2.11)}$$

The total spinning-reserve constraint can, then, be written as;

$$\sum_i S_{ij} \ge ST_j \quad \forall_{i,j} \qquad \text{... (2.12)}$$

Transmission-network constraints and station limits may be expressed by a number of group import-export constraints;

$$I_g \le \sum_{i \in g} G_{ij} \le E_g \quad \forall_{j,g} \qquad \text{... (2.13)}$$

Solution Scheme

An efficient solution of the overall model presented above can be obtained by applying Dantzig-Wolfe decomposition [3] (see appendix). The basic model would, then, constitute a transshipment subproblem solved by the network Simplex method with the additional reserve and import-export group constraints being the linking constraints used to form the master problem.

It is well known that the number of linking constraints has a decisive effect on the computational efficiency of Dantzig-Wolfe decomposition. In the present model, the constraints representing the reserve contribution of each generators, i.e. constraints (2.10) and (2.11), constitute the majority of the linking constraints. Fortunately, a drastic reduction of their number proved possible.

A number of case studies revealed that the optimal dispatch is very close to that obtained if the reserve constrints are neglected. Therefore, the effect on the optimality of the solution would be negligible, if any, if the following strategy is implemented. The problem is solved first by Dantzig-Wolfe decomposition with the group constraints only. The reserve constraints are, then checked; if any is violated, the region of operation of each generator during each period is fixed according to the current solution and constraints (2.10), (2.11) and (2.12) are substituted by;

$$\sum_{i} (\lambda_{ij} k_i G_{ij} + (1 - \lambda_{ij})(G_i^M - G_{ij})) \geq ST_j \quad \forall_{i,j} \qquad \ldots (2.10)$$

where

$$\lambda_{ij} = \begin{cases} 1 \text{ if generator i is operating In the lower region} \\ 0 \text{ otherwise} \end{cases}$$

Thereafter, the Dantzig-Wolfe iterations are resumed starting from the current solution.

It is also well known that the convergence of Dantzig-Wolfe decomposition can be slow in the absence of an appropriate stopping criterion [9]. In the present context, the value of the objective function at the optimal solution of the basic model, without the additional linking constraints, constitutes a lower bound on the optimal value of the overall problem. Hence, if a solution satisfying all constrainst is found with a cost sufficiently close to the lower bound, calculations can be terminated and his solution can be safely considered optimal.

Thus the overall solution procedure can be described as follows;

Step 1. Solve the basic transshipment model. Calculate the lower bound cost.
Step 2. Form the linking constraints corresponding to the group import-export limits (2.13)
Step 3. Start Dantzig-Wolfe iterations until a solution feasible in the group import-export is found.
Step 4. Fix the region of operation of each generator during each period to correspond to that indicated by the solution in step 3. Form the linking constraints (3.14).
Step 5. Resume Dantzig-Wolfe iterations, starting from the solution found in step 3. Stop when a solution feasible in all constraints is found with a cost sufficiently close to the lower bound cost.

It is worth noting that in view of the logical structure of the linking constraints, they are not explicitly assembled and stored; rather they are operated on implicity when forming the columns of the master problem. This has the effect of enhancing computational efficiency.

Constraint relaxation

The decomposition described above achieves natural division of constraints into two groups: the hard constraints included in the transshipment model and the soft linking constraints. This makes the proposed dispatch method suitable for use within a constraint relaxation framework. In cases where the original dispatch problem proves infeasible, it is very desirable to be able to relax one or more of the soft constraints. This can be achieved in the present context by dividing the soft constraints into a hierarchy from softer to harder. If the number of Dantzig-Wolfe interations exceeds a pre-set limit, this is taken to indicate infeasibility and the following constraints relaxation steps are taken interatively;

Step i. Identify the constraints violated by the current solution.
Step ii. Choose the most violated of the softest group for relaxation and relax it by a certain pre-set percentage.
Step iii. Update the solution of the master problem.
Step iv. Proceed with Dantzig-Wolfe iterations. If a feasible solution with a cost sufficiently near that of the lower bound solution is found, stop, else if a pre-set number of Dantzig-Wolfe iterations is reached, go to step i.

Case Studies

Two versions of the optimisation model were developed in PASCAL on a SUN 3/50 microsystem. The two versions differ in the treatment of spinning reserve constraints. In the first, these constraints were represented in full, i.e. as (2.10) and (2.12). In the second, the regions of operation of generators were fixed, as in step 4 above, and (3.14) used to enforce the reserve requirements.

Case Study 1

The relative performance of the two versions of the model was studied on a number of examples, without group constraints. The results in table 3 are representative. They show that the second version is much more efficient, being faster by an order of magnitude, with a negligible difference in dispatch cost; well within round-off error tolerance.

Table 3 Comparison between the two versions of model (for 6 periods)

Generators	Version cost (units)	Generation lower bound %	Margin over time (sec)	Total run
20	1	27035.11	0.07	204.2
	2	27028.43	0.04	10.9
30	1	116161.41	0.22	347.0
	2	116204.41	0.25	29.4

The computational efficiency of t he algorithm is due to its rapid covergence. For 6 periods and 20 generators, convergence required 9 Dantzig-Wolfe iterations, while for 30 generators, 12 such iterations were required. Convergence is tested for using the stopping criterion of proximity to the lower bound solution. In all test cases, the first feasible solution obtained satisfied this criterion by a very close margin and was deemed optimal. The margins were calculated as: (cost of solutiont – lower bound cost)/lower bound cost.

The variation of run time with the number of periods in the dispatch horizon was also studied. Even though the number of periods would, probably, be between 4 and 6 In an operational setting, it was increased up to 24 periods.The result, shown in table 4, domenstrates that the increase in run-time is moderate.

Table 4 Variation of run time with number of periods for the 20 generators system

Periods	4	8	16	24
Run time(sec.)	3.8	17.5	117.6	312.0

Case Study 2

The low computational requirements of the proposed algorithm make it suitable for on-line dynamic dispatch for large-scale power systems. This has been validated by a series of tests on modified data taken from [6]. The system comprises 100 units and the horizon consists of 5 periods. Transmission network and station-limit constraints are imposed by grouping the generators into 22 groups and placing limits on group power import and export. The tests were carried out by varying the demand profile and reserve requirements to correspond to periods intervals of a typical demand cycle. Tables 5, 6 and 7 present system data and table 8 present the results. Typically solution was achieved in 330 seconds approximately on a SUN 3/50 microsystem.

Table 5 Generator data for case study 2 (100 generators)

Generator number	Generator Gmi(MW)	limits GMi(MW)	SL (MW)	Cost (Units)
1	10	60	55.0	19.0
2	10	60	55.0	19.0
3	10	60	55.0	20.0
4	10	60	55.0	20.0
5	10	60	55.0	20.0
6	20	100	90.0	20.0
7	20	100	90.0	20.0
8	50	500	0	15.0
9	50	500	0	15.0
10	50	500	0	15.0
11	50	500	0	15.0
12	10	60	55.0	19.0
13	10	60	55.0	19.0
14	10	60	55.0	19.0
15	20	100	90.0	20.0

Table 5 Generator data for case study 2 (100 generators)

Generator number	Generator Gmi(MW)	limits GMi(MW)	SL (MW)	Cost (Units)
16	20	100	90.0	20.0
17	20	100	90.0	19.0
18	20	100	90.0	19.0
19	50	100	94.0	20.0
20	20	100	90.0	20.0
21	20	100	90.0	20.0
22	20	100	90.0	20.0
23	10	60	55.0	21.0
24	20	50	48.0	22.0
25	10	40	38.0	22.0
26	30	60	56.0	21.0
27	10	50	46.0	20.0
28	10	50	46.0	20.0
29	10	60	55.0	19.0
30	10	60	55.0	19.0
31	10	60	55.0	19.0
32	20	100	90.0	20.0
33	20	100	90.0	20.0
34	20	100	90.0	20.0
35	20	100	90.0	20.0
36	20	100	90.0	20.0
37	10	50	46.0	19.0
38	10	50	46.0	19.0
39	10	50	46.0	20.0
40	10	50	46.0	20.0
41	10	50	46.0	20.0
42	20	50	48.0	19.0
43	10	50	46.0	19.0
44	10	50	46.0	19.0
45	20	50	48.0	21.0
46	20	50	48.0	22.0
47	10	60	55.0	19.0
48	10	60	55.0	19.0
49	10	60	55.0	19.0
50	10	60	55.0	19.0
51	5	30	28.0	22.0
52	5	30	28.0	22.0
53	5	30	28.0	22.0
54	5	30	28.0	22.0
55	5	30	28.0	22.0
56	10	60	55.0	20.0
57	10	60	55.0	20.0
58	10	60	55.0	20.0
59	20	50	48.0	21.0
60	20	50	48.0	21.0
61	20	50	48.0	21.0
62	30	50	48.0	21.0
63	20	50	48.0	21.0
64	10	50	46.0	21.0
65	20	50	48.0	21.0
66	20	100	90.0	18.0
67	20	100	90.0	18.0
68	10	60	55.0	20.0
69	10	60	55.0	20.0
70	10	60	55.0	20.0
71	10	60	55.0	20.0
72	10	60	55.0	20.0
73	10	50	46.0	19.0
74	10	50	46.0	21.0
75	10	50	46.0	21.0
76	10	50	46.0	21.0
77	10	50	46.0	21.0
78	20	60	55.0	20.0
79	20	60	55.0	20.0

Table 5 Generator data for case study 2 (100 generators)

Generator number	Generator Gmi(MW)	limits GMi(MW)	SL (MW)	Cost (Units)
80	10	50	46.0	19.0
81	50	500	0	15.0
82	40	400	0	16.0
83	50	500	0	15.0
84	10	50	46.0	20.0
85	10	50	46.0	19.0
86	10	50	46.0	19.0
87	10	50	46.0	19.0
88	10	50	46.0	19.0
89	10	40	38.0	19.0
90	20	60	55.0	20.0
91	20	60	55.0	20.0
92	10	50	46.0	20.0
93	20	60	55.0	20.0
94	10	50	46.0	22.0
95	10	50	46.0	22.0
96	30	50	48.0	21.0
97	20	50	48.0	22.0
98	20	50	48.0	22.0
99	20	50	48.0	22.0
100	20	50	48.0	22.0

Table 6 Data for case study 2 (100 generators)

Group Ig(MW)	limits Eg(MW)	Generators in group							
40	250	1,	2,	3,	4,	5			
40	200	6,	7						
100	1500	8,	9,	10,	11				
20	160	12,	13,	14					
140	750	15,	16,	17,	18,	19,	20,	21,	22
40	200	23	24	25	26				
20	2000	27,	28						
10	450	32,	33,	34,	35,	36			
10	190	37	38,	39,	40				
10	150	45,	46						
40	200	47,	48,	49,	50				
10	160	51,	52,	53,	54,	55			
30	200	56,	57,	58					
100	300	59,	60,	61,	62,	63,	64,	65	
40	150	66,	67						
10	280	68,	69,	70,	71,	72			
50	200	73,	74,	75,	76,	77			
50	180	78,	79,	80					
120	1200	81,	82,	83					
60	6000	84,	85,	86,	87,	88,	89		
20	4000	90,	91,	92,	93				
100	200	96,	97,	98,	100				

Table 7 Data for case study 2 (100 generators)

Period	1	2	3	4	5
Demand (MW)	7000	7500	7250	7700	7100
Required reserve (MW)	240	240	240	240	240

Table 8 Results for case study 2(100 generators)

Dispatch (MW) Period Generator	1	2	3	4	5
1	58.0	58.1	58.1	54.4	58.1
2	58.0	58.1	58.1	54.4	58.1
3	35.5	42.4	43.5	46.8	42.0
4	35.5	42.4	42.9	46.8	42.0
5	45.7	43.6	41.9	47.7	49.2
6	90.0	97.4	95.4	M	81.0
7	90.0	M	96.4	M	77.3
8	378.0	406.1	361.4	322.8	360.3
9	366.1	338.3	367.8	322.8	360.3
10	378.0	354.0	367.8	358.0	367.0
11	378.0	401.6	403.0	496.3	377.3
12	50.1	48.3	48.3	48.3	48.3
13	50.1	48.3	48.3	48.3	48.3
14	50.1	48.3	48.3	48.3	48.3
15	64.0	97.4	89.4	91.0	75.6
16	57.8	96.3	83.0	91.0	67.3
17	M	M	M	97.0	M
18	M	M	M	97.0	M
19	73.6	97.3	88.2	98.1	75.3
20	57.8	84.6	80.9	94.1	60.5
21	57.8	84.2	63.5	93.4	60.5
22	57.8	90.2	63.5	88.4	60.5
23	31.5	36.2	32.6	41.0	35.3
24	29.3	35.7	33.5	37.7	31.1
25	19.3	25.7	23.5	27.7	25.2
26	42.9	45.7	43.6	50.0	45.2
27	28.9	35.6	31.8	45.1	39.7
28	28.9	35.6	31.8	45.1	39.7
29	M	M	M	M	M
30	M	M	M	M	M
31	M	M	M	M	M
32	55.8	71.1	63.5	82.6	79.4
33	60.4	71.1	69.8	82.6	79.4
34	60.6	80.8	72.9	92.4	79.4
35	60.6	80.8	79.4	92.4	79.4
36	66.8	80.8	75.3	M	79.4
37	M	M	M	45.4	M
38	M	M	M	45.4	M
39	33.6	40.5	37.6	45.4	39.7
40	33.6	49.9	37.6	45.4	39.7
41	33.6	M	37.6	M	46.2
42	M	M	M	M	M
43	M	M	M	M	M
44	M	M	M	M	M
45	32.9	35.7	33.6	40.0	35.2
46	29.3	35.6	33.5	40.0	29.3
47	50.0	50.0	48.1	50.9	50.8
48	50.0	50.0	51.9	50.9	49.7
49	50.0	50.0	51.9	50.9	49.7
50	50.0	50.0	48.1	47.2	49.7
51	12.7	17.2	16.3	21.6	12.8
52	12.7	17.0	16.3	21.6	12.8
53	12.7	17.0	16.3	21.6	12.8
54	12.7	17.0	16.3	21.6	12.8
55	12.7	17.0	13.9	21.6	12.8
56	58.0	58.6	55.4	M	M
57	58.0	58.0	55.4	M	M
58	58.0	58.0	55.4	M	M
59	32.8	35.7	33.6	40.0	35.2
60	32.8	35.7	33.6	40.0	35.2
61	32.8	35.7	33.6	40.0	35.2
62	38.6	40.5	39.1	43.3	40.1
63	32.8	35.7	33.6	40.0	35.2

Table 8 Results for case study 2(100 generators)

Dispatch (MW) Period Generator	1	2	3	4	5
64	27.1	30.9	28.1	36.6	30.2
65	32.8	35.7	33.6	40.0	35.2
66	72.1	72.1	72.1	72.1	67.6
67	72.1	72.1	72.1	71.7	67.6
68	56.0	56.0	5.4	M	58.1
69	56.0	56.0	55.4	56.0	58.1
70	56.0	56.0	55.4	54.3	58.1
71	56.0	56.0	55.4	54.3	58.1
72	56.0	56.0	55.4	54.3	46.3
73	M	M	M	M	M
74	27.1	30.9	28.1	36.6	30.2
75	27.1	30.9	28.0	36.6	30.2
76	27.1	30.9	28.0	36.7	30.2
77	22.9	30.9	28.0	36.7	30.2
78	58.4	58.4	56.3	55.4	50.5
79	58.4	58.4	56.3	55.4	50.5
80	M	M	M	M	M
81	428.6	427.3	428.6	428.4	412.2
82	342.9	341.8	342.9	315.6	329.8
83	428.6	430.9	428.6	456.0	412.2
84	48.4	48.4	46.3	45.4	40.5
85	M	M	M	M	M
86	M	M	M	M	M
87	M	M	M	M	M
88	M	M	M	M	M
89	M	M	M	M	M
90	58.4	58.4	56.3	55.4	50.5
91	58.4	58.4	56.3	55.4	50.5
92	48.4	48.4	46.3	45.4	40.5
93	58.4	58.4	56.3	55.4	50.5
94	22.3	29.3	23.2	36.6	22.4
95	22.3	29.3	23.2	36.6	22.4
96	40.6	40.5	39.0	43.4	40.1
97	L	34.4	29.9	38.9	29.3
98	L	34.4	29.9	38.9	29.3
99	L	34.4	29.9	38.9	29.3
100	L	34.4	29.2	38.9	23.9

Symbols are as follows;
M : Unit generating at maximum power
L : Unit generating at minimum power

It would have been interesting to compare the efficiency of the proposed algorithm with that of Irving and Sterling [6]. However, this does not seem to be possible in ameaningful way due to the different characteristics of the computers employed. Moreover, it appears that reserve constraints were not included in the computational experiment reported in [6] leading to a total of 2225 constraints compared to 3230 constraints in the present case study.

An important feature of the proposed algorithm is that solution time varies polynomialy as the number of generators increases, with a modest increase law. This is so because the solution time for a transshipment problem, solved by the Network Simplex method, is much less sensitive to problem dimension than an ordinary linear programming problem, solved by the Revised Simplex method. As a result of the decomposition scheme, the number of generators affects only the size of the transshipment problem, leaving the number of linking constraints unchanged except if a small number of group constraints is added.

Conclusions

Formulation of mathematical programming model for the secure, multi-period dynamic economic dispatch problem has been presented. By exploiting the structural properties of the model, an efficient solution scheme based on Dantzig-Wolfe decomposition has been developed. The efficacy and efficiency if the model and solution scheme have been demonstrated By case studies. The method is believed to be suitable for on-line dispatch of large power systems and can be used within a constraint relaxation framework.

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Appendix

Dantzig-Wolfe Decomposition Principle

Consider the linear-programming problem

min cx

subject to

$$Ax = b;\ 0 \le x \le u \qquad \ldots (A.1)$$

The matrix A(m × n) can be decomposed into two matrices A'(m' × n) and matrix A"(m" × n). Thus, (A.1) can be written as

min cx

subject to

$$A'x = b' \qquad \ldots (A.2)$$
$$A''x = b''$$
$$0 \le x \le u \qquad \ldots (A.2)$$

Let the subproblem

min *x

subject to

$$A''x + b''$$
$$0 \le x \le u \qquad \ldots (A.3)$$

have basic feasible solution $v^1, v^2, \ldots, v^M$. Then a vector x will satisfied (A.3) if and only if (Chvatal [3])

$$x = \sum_k \lambda_k v^k \qquad \ldots (A.4)$$

where λ_k are non-negative numbers such that $\sum\lambda_\kappa = 1$. Substituting (A.4) into (A.2) yields the following so-called master problem

$$\min \textstyle\sum_k f_k \lambda_k$$

subject to

$$\textstyle\sum_k g_k \lambda_k = b'$$
$$\textstyle\sum_k \lambda_k = 1$$
$$\lambda_k \ge 0 \qquad \ldots (A.5)$$

where $f_k = cv^k$ and $g_k = A'v^k$.

The solution of original problem is obtained by first solving the subproblem to generate the f_k's and g_k's and then the master problem, iteratively until optimality is reached.

The subproblem can be written as

$$\min (c - \pi' A')x$$

subject to

$$A''x + b'';\ 0 \le x \le u \qquad \ldots (A.6)$$

where π' are the simplex multipliers (or dual variables) obtained from the solution of master problem.

After solving the subproblem a relative cost factor is calculated by

$$f_k^* = (c - \pi' A')v - \pi_o \qquad \ldots (A.7)$$

where π_o is the simplex multiplier associated with the subproblem. If all f_k^* are positive, the optimality has been reached.

If not optimum, then generate a column

$$\bar{a} = \begin{bmatrix} A'v \\ 1 \end{bmatrix} \qquad \dots \text{(A.8)}$$

with the corresponding component of c equal to cv.
The optimal vector x is obtained by using (A.4).