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COMPUTERISED CALCULATIONS FOR WATER LEVEL AND VELOCITY VARIATIONS IN THE SURGE TANK SYSTEM

by

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Abstract

Water level variations in the surge tank and velocity variation in the pressure conduits in a surge tank system can be computed by using the finite difference solution of the governing differential equations. A computer program that was written in Fortran 77 is based on the solution technique. A laboratory experiment in determining the variations of water level in a surge tank was conducted to complement the development of the computer program. The results of the calculated and recorded surge tank water level variations are compared and a goodness of a regression curve is expected.

Introduction

Surge tank is an open standpipe connected to a conduit or tunnel from a reservoir of a hydroelectric power plant or to the pipeline of a water supply piping system (Jaeger, 1977). Sometimes surge tank is also called surge shaft or surge chamber. Surge tanks are constructed in hydroelectric power plant or in water supply piping system for various purposes and functions.

The main function of surge tank constructed in both hydroelectric power plant and in water supply piping system is to reduce the amplitude of pressure fluctuations in conduit, tunnel or pipeline by reflecting the incoming pressure waves (Chaudhry, 1979). Pressure waves caused by water hammer effects in a penstock due to sudden control valve closure or load change in turbine are reflected back at the surge tank. If there is no surge tank present in the system, the pressure waves with very high pressure and thus cause the conduit to burst. Water hammer effect can be reduced and it occurs only between the turbine or the control valve and the surge tank rather than between the turbine/control valve and reservoir. Placing the surge tank will certainly reduce the pressure variations in the pressure conduit. The conduit strength can also be reduced since the pressure is now less than the pressure when the surge tank is not provided because water hammer gives very high pressure.

The other function of surge tank is that the head in the surge tank imposes regulating characteristics of a hydraulic turbine (Chaudhry, 1979). The water-starting time (to initially run the turbine) is nor from the head at the surge tank rather than from the far upstream reservoir. Therefore the water-starting time of a hydroelectric power plant is reduced and thus improving the regulating characteristics of the power plant.

The third function of surge tank in the system is that it acts as a storage of excess water during the hydraulic turbine load reduction in a hydroelectric power plant (Chaudhry, 1979). It also provides water during the hydraulic turbine load increment in the power plant. In both cases the water in the pressure pipeline of penstock is accelerated or decelerated gradually and thus the amplitude of pressure fluctuations in the system is reduced.

The design of a surge tank is depended on the type of the surge tank itself. For simple surge tank, the height is designed based on the maximum water level after sudden closure of the control valve. This maximum water level will depend on the tank horizontal cross sectional area, the conduit cross sectional area, the initial flow rate and the type of the simple tank whether it is throated or not.

Theoretial Considerations

Let us consider the hydraulic surge tank system as shown in Figure 1.

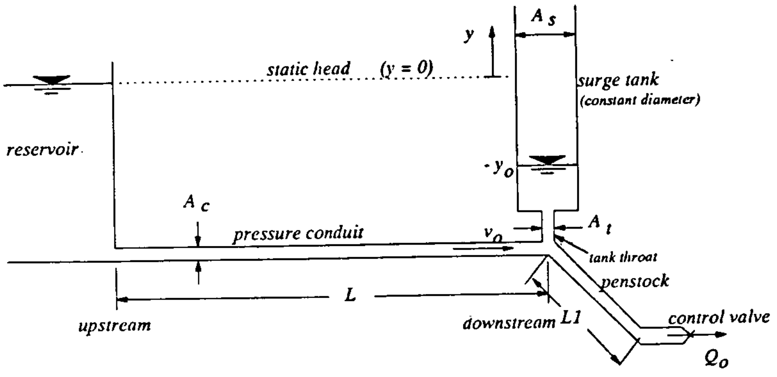
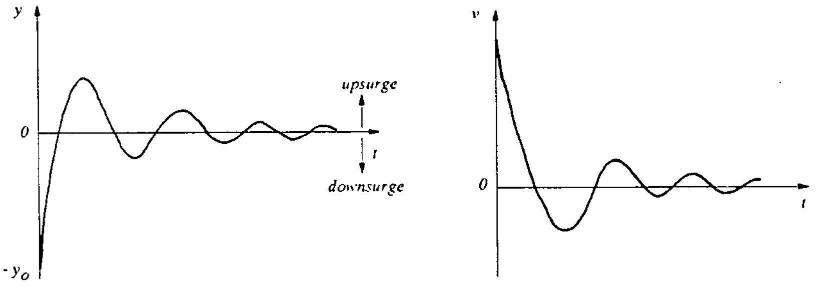


Figure 1: Definition diagram of the surge tank system

The pressure conduit of length L is very sensitive to pressure variation. The surge tank is constructed to protect the pressure conduit from high pressure variation by reflecting pressure wave travelling up the penstock after the closure of the control valve. The pressure pipeline or the penstock of length L_l is constructed with sufficient strength to withstand high pressure from sudden control valve closure. Thus the surge tank reduces the pressure oscillation in the penstock.

During the opening of the control valve, the water level in the surge tank is below the static head or the reservoir surface level due to head loss in the pressure conduit. During this time the water flows in steady state condition with constant flow rate of Q_o and with constant positive velocity v_o Control valve closure will cause the pressure wave to travel upstream in the penstock up the surge tank. At the same time the water in the pressure conduit is still downstream to the surge tank. This will cause the water level in the surge tank to rise above the static head. Since the water level in the level in the surge tank is much higher than the reservoir water level, water flows from the surge tank to the reservoir and thus the velocity becomes negative. This will occasionally reduce the water level in the surge tank. The flowing water travel back from the reservoir to the surge tank since the water level in the surge tank is much lower then the reservoir water level. This will cause the water level in the surge tank to rise again but lesser height compared to the first one. This phenomena is called mass oscillation.

Water level variation (in time) in the surge tank and velocity variation in the surge tank and velocity variation in the pressure conduit near the surge tank can be predicted. The governing equations are transformed to a computer program so that the calculation for the water level variation be calculated in no time. The results are plotted and they are called the mass oscillation curves that produces the pattern as shown in Figure 2.



(a) (b)

Figure 2: Mass oscillation curves

The Governing Equations

The governing equations for the water level variation (in time) in surge tank (y as a function of time, t) and for the velocity variation in pressure conduit (υ as a function of time, t) are derived based on the assumption that (Jaeger, 1977)

1. the pressure conduit and penstock wall are rigid,
2. water is incompressible,
3. the steady state flow frictional resistance at any time can be used,
4. the surge tank is vertical with constant horizontal cross sectional area, and
5. the surge tank wall is frictionless.

The governing equations are described by the *dynamic equation* and the *continuity equation.* Consider the surge tank system as shown in Figure 1, which is the definition diagram. The objective of the equation is to obtain the mass oscillation functions as shown in Figure 2. The two governing equations are as follows.

Dynamic Equation

The dynamic equation is derived based on the element of length of the pressure conduit (Jaeger, 1977). The forces acting on the element are the component of water weight, the pressure force and the frictional resistance force. If Newton's first law of motion is applied to one-dimensional flow through the element of water, the dynamic equation can be written as

$$\frac{L}{g}\,\frac{dv}{dt} + y + F_p\, v\,|v| + + \; F_t\, u\,|u| = 0 \qquad (1)$$

where

v = flow velocity in pressure conduit (m/s)
t = time is second
y = water level in surge tank above the static level (m)
u = upward water velocity in surge tank (m/s)

$$F_p = -\frac{y_o}{v_o^2} \qquad (2)$$

y_o = initial surge tank water level (m)

$$f_t = \frac{K_t}{2g}\,\frac{A_s}{A_t} \qquad (3)$$

K_t = head loss coefficient for surge tank throat

Continuity Equation

The continuity equation is derived based on the theory of conservation of mass in a control volume in the junction of the pressure conduit, surge tank and the penstock (Parmakian, 1955). It can be written as

$$u = v \frac{A_c}{A_s} \tag{4}$$

and

$$dy = u\,dt \tag{5}$$

Solution Using Step-by-Step Integration

The governing differential equations (Eq.1, 4 and 5) are in the form of water level (y) and velocity (v) as a function of time (t). The solution of these equations is to determine the water level and velocity variation (in time) so that all water level and velocity values be calculated at any time, t after the control valve closure. To determine these values, the governing equations are solved by replacing them by using difference equation (Jaeger, 1977). This means that the infinitesimally small time interval dt is replaced by a small,but finite, interval Dt. The finite-difference equations equations are then solved using step-by-step integration method. The finite-difference equation of Eq. 1 is written as

$$\frac{L}{g}\frac{\Delta v_i}{\Delta t} + y_i + F_p v_i |v_i| + F_t\, u_i |u_i| = 0$$

which can be rewritten as

$$\Delta v_i = -(y_i + F_p\, v_i |v_i| + F_t\, u_i\, |u_i|)\frac{g\Delta t}{L} \tag{6}$$

where i = time index (i = 1,2,3,4,...,i_{max})

The finite-difference of Eq. 4 and 5 are

$$u_i = v_i \frac{A_c}{A_s} \tag{7}$$

and

$$\Delta y_i = u_i\, \Delta t \tag{8}$$

respectively. The step-by-step integration method of solution used the following additional continuity equation that is

$$v_i = v_{i-1} + \Delta v_{i-1} \tag{9}$$

and

$$y_i = y_{i-1} + \Delta y_i \tag{10}$$

Initial Condition

The initial condition is for time $t = 0$ or $i = 1$ which indicates all values and information known in the surge tank system before the closure of the control valve. The initial values for the velocity and velocity change (Dv) in the pressure conduit, the vertical velocity in surge tank and the water level and water level change (Dy) are as follows.

1. $v_1 = v_o = Q_o/A_c$

2. $Dv_1 = 0$

3. $u_1 = v_o A_c/A_s$

4. $y_1 = y_o$

5. $\Delta y_1 = 0$

These initial values must be given before any calculations using the finite-difference equations can be performed. Besides these initial values, the following input data must also be given which are

1. time interval, Δt (the smaller the better),
2. surge tank horizontal cross-sectional area, A_s,
3. pressure conduit cross sectional area, A_c,
4. throated tank horizontal cross sectional area, A_t,
5. pressure conduit length, L,
6. gravitational acceleration, g,
7. initial surge tank water level, y_o, .
8. initial flow rate, Q_o and
9. head loss coefficient of surge tank throat, K_t

The Computer Program

The calculation of water level and velocity variation in time in a surge tank system is based on the finite-difference equation (Eq. 6 to 10) repeated for every time step. For convenience and best result, the calculations have to be performed by using a computer program. A computer program was written in Fortran 77 utilizing the IBM-PC compatible microcomputers. Figure 3 shows the flowchart which outlines the procedure for solving the system of the difference equations.

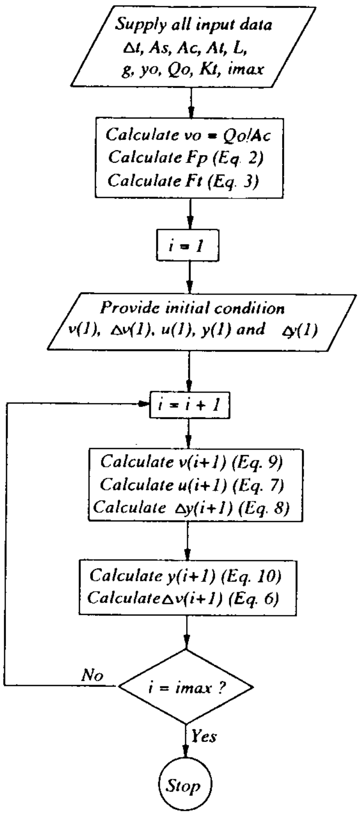


Figure 3: Flowchart for solving the difference equations of the surge tank system

Program Applications

The computer program is applied to a real water variation data obtained from a laboratory experiment. A surge tank system equipment is available in the Hydraulics Laboratory, Faculty of Civil Engineering. Water level variation in a 44 mm diameter surge tank was recorded after the closure of the control valve. The surge tank system data information and dimensions are collected so that they can be used as the input data to the computer program. The program is run and the results of water level variation are plotted and are called calculated water level variations. The calculated water level variations are compared to the recorded water level variations and the regression coefficient of determination, r^2 is determined.

The dimensions of the surge tank equipment and the initial data collection are as follows.

1. Initial surge water level, y_o = -585 mm,
2. Initial flow rate, $Q_o = 4.478 \times 10^{-4}\ \mathrm{m}^3/\mathrm{s}$
3. Pressure conduit length, L = 3.0 m
4. Pressure conduit diameter, D_c = 20.2 mm
5. Surge tank diameter, D_s = 44 mm
6. Diameter of surge tank throat, D_t = 20.2 mm
7. Time interval for calculation, Δt = 0.2 seconds

Results and Conclusion

The computer running time was several seconds (not more than 5 seconds) on IBM-PC AT 386 microcomputers. The output file is shown in the Appendix. Figure 4 shows the variation of velocity in the pressure conduit near the surge tank. Figure 5 shows the water level variation in the surge tank. Two curves of graph water level (y) versus time (t) are plotted. The first curve indicates the calculated values (using computer program) of water level versus time while the second curve indicates the recorded water level (experiment values) of water level versus time. The calculated and recorded values of water level versus time are compared by determining the regression coefficient of determination, r^2 which is

$$r^2 = \frac{\Sigma(y_{recorded} - y_{mean,\ recorded})^2 - \Sigma(y_{calculated} - y_{recorded})^2}{\Sigma(y_{recorded} - y_{mean,\ recorded})^2} \tag{11}$$

which gives the value of $r^2 = 0.87$. From this r^2 value, the author concluded that the calculated and recorded water level in surge tank is high correlated. This means that the theory outlined which is summarized by Eq.1 - 5 provides reasonable predictions of the variation of water level in surge tank as well as the variation of velocity in the pressure conduit.

References

(1) Chaudhry, H.M. (1979), Applied Hydraulic Transients. Van Nostrand Peinhold Publishing Co., New York.

(2) Jaeger, C. (1977), Fluid Transients in Hydro-electric Engineering Practise Blackie & Son Publishers Ltd., London.

(3) Parmakian, J. (1955), Water Hammer Analysis. Prentice Hall, New York.

Figure 4 : Velocity Variation in pressure conduit

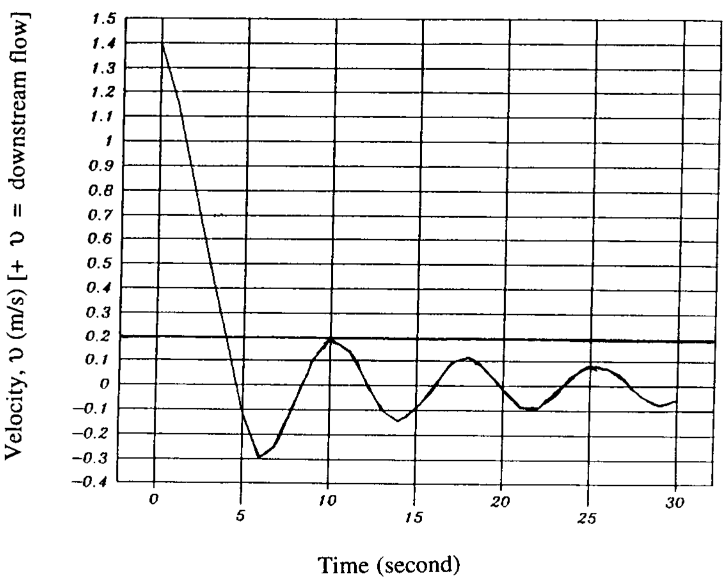
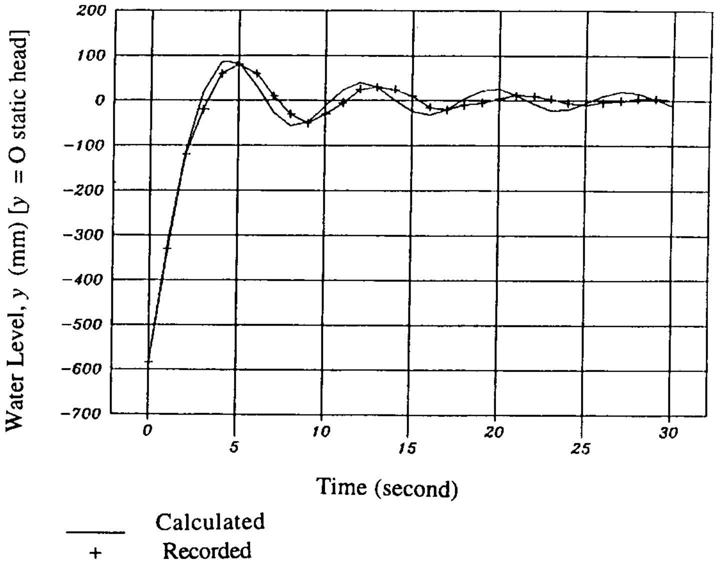


Figure 5 : Water level variation in surge tank



APPENDIX

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      SSSS   TTTTTTT     A      NN    N    K   K
     S          T       A A     N N   N    K  K
      SSS       T      A   A    N  N  N    KK
         S      T      AAAAA    N   N N    K  K
    SSSSS       T      A   A    N    NN    K   K

*>>>>>>>>>   SURGE TANK COMPUTER PROGRAM   <<<<<<<<<*
* (Calculates  water level and velocity variation)  *
*                                                   *
*                       by                          *
*                AMAT SAIRIN DEMUN                  *
*          Universiti Teknologi Malaysia            *
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Type of Tank   :  Constant Diameter Throated Tank
Project Name   :  LABORATORY EXPERIMENT ON SURGE TANK SYSTEM
User's Name    :  Amat Sairin Demun
Organization   :  Universiti Teknologi Malaysia
File Name      :  A:LABSURG.DAT

DATA OF THE SURGE TANK SYSTEM :-
Datum = Static Head =  0.0 m
Initial surge tank water level, yo =  -0.585 m
Initial flowrate, Qo =  0.0004478 m3/s
Pressure conduit length,  L =  3.000 m
Pressure conduit diameter,  Dc =  0.0202 m
Surge tank diameter,  Ds =  0.044 m
Time interval for calculation,  Dt =  0.20 saat
Diameter of surge tank throat,  Dt =  0.0202 m
Head loss coefficient of surge tank throat,  Kt =  1.0


CALCULATION TABLE
---------------------------------------------------------
    t        Dv         Dy        v         u         y
 (sec)     (m/s)      (m)       (m/s)     (m/s)      (m)
---------------------------------------------------------
    0.0     0.000     0.000     1.397     0.295    -0.585
    1.0    -0.065     0.049     1.156     0.244    -0.315
    2.0    -0.066     0.035     0.828     0.175    -0.113
    3.0    -0.065     0.021     0.502     0.106     0.021
    4.0    -0.063     0.008     0.182     0.038     0.086
    5.0    -0.053    -0.005    -0.124    -0.026     0.085
    6.0    -0.004    -0.012    -0.294    -0.062     0.033
    7.0     0.028    -0.010    -0.240    -0.051    -0.025
    8.0     0.037    -0.003    -0.075    -0.016    -0.055
    9.0     0.029     0.004     0.106     0.022    -0.048
   10.0     0.000     0.008     0.195     0.041    -0.012
   11.0    -0.021     0.006     0.147     0.031     0.025
   12.0    -0.027     0.001     0.025     0.005     0.041
   13.0    -0.018    -0.004    -0.100    -0.021     0.030
   14.0     0.003    -0.006    -0.146    -0.031     0.001
   15.0     0.018    -0.004    -0.096    -0.020    -0.024
   16.0     0.021     0.000     0.004     0.001    -0.032
   17.0     0.011     0.004     0.095     0.020    -0.019
   18.0    -0.006     0.005     0.115     0.024     0.005
   19.0    -0.016     0.003     0.062     0.013     0.023
   20.0    -0.017    -0.001    -0.023    -0.005     0.026
   21.0    -0.006    -0.004    -0.089    -0.019     0.012
   22.0     0.007    -0.004    -0.091    -0.019    -0.008
   23.0     0.014    -0.002    -0.037    -0.008    -0.022
   24.0     0.013     0.001     0.036     0.007    -0.020
   25.0     0.003     0.003     0.082     0.017    -0.006
   26.0    -0.008     0.003     0.071     0.015     0.011
   27.0    -0.013     0.001     0.018     0.004     0.020
   28.0    -0.010    -0.002    -0.044    -0.009     0.016
   29.0     0.000    -0.003    -0.074    -0.016     0.002
   30.0     0.009    -0.002    -0.055    -0.012    -0.012
---------------------------------------------------------
ABBREVIATIONS :
t  = time after control valve closure
v  = flow velocity in pressure conduit
Dv = Change in flow velocity in pressure conduit
y  = Water level in surge tank above the static head
Dy = Water level change in surge tank
u  = Upward water velocity in surge tank
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