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## NON-LINEAR STRESS-STRAIN MODEL FOR COATED FABRICS

by

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### ABSTRACT

Coated fabrics are used in a variety of types and strengths for prestressed membrane structures. To account properly for their behaviour more accurate numerical models than those in common use have to be developed. A program was written to calculate the stresses in a uniform stress, triangular element based on the model using Dynamic Relaxation Method. This model is applied to experimental results developed from a sample of coated fabrics and comparisons are made between experimental results and model predictions. The model is numerically simple and well suited to implementation in numerical schemes necessary for non-linear analysis of complete structures.

### INTRODUCTION

Coated fabrics function as a major load-resisting component in pneumatic structures, tents, and net structures as well as in numerous industrial applications. Even though these types of structure have been erected in many parts of the world, the analysis, design and construction of fabric structures remain essentially skills mastered by comparatively few practitioners.

The increased use of coated fabrics in recent years has focused attention on the need for suitable models to be used in analyses where biaxial states of stress prevail. Designers are confronted by a material that not only exhibits a nonlinear response in uniaxial tension but also nonlinear anisotropic behaviour and, more importantly, shows apparently anomalous behaviour under biaxial loading.

The use of linear orthotropic stress-strain relation as an approximation to nonlinear stress-strain curves for fabric at this stage is not a good approach. So it is necessary to develop a stress-strain model which will accurately represent the behaviour of coated fabric.

Previous studies[1,2,3] show that several models for coated fabrics have been proposed to explain the biaxial mechanical response of the materials. Although many of these works have produced a good agreement between theoretical and experimental results, most are not formulated in a manner which is well suited to implementation in numerical schemes necessary for nonlinear analysis of complete fabric structures.

### **BEHAVIOUR OF COATED FABRIC**

A coated woven fabric is considered anisotropic because of the behaviour caused by its interstructure as well as the coating condition of the fabric (see Figure 1). The peculiarities of fabric response to applied loads may be attributed to the deformation of one or more mechanisms dominating in certain ranges of loading.

Some of the mechanisms that govern the deformation of such fabrics include crimp interchange between warp and weft, bending and torsion of yarns, the load extension of yarns, crushing of yarns at crossover points, friction between yarns at point of contact, bending of coating and interaction between the fabric and coating material.

### **BIAXIAL TEST CURVE**

Typical curves from the result of biaxial tests for PTFE-glass fabric are shown on Figure 2. On it, the curve obtained from tests in which equal tensions are applied to the warp and weft, are shown as the curves for biaxial 1:1. The curves 5:1 and 1:5 are for these ratios of warp to weft stresses. These curves show the characteristics of PTFE-glass fabric. In the biaxial 1:1 curves the nearly horizontal part, seen in the weft curve, represents the behaviour of the fabric when the crimp is straightened out. The curves show that little force is required to do this.

### **NUMERICAL MODEL**

#### **General Equations.**

It is evident that to correctly reproduce test curves in calculations a non-linear set of equations is required. When investigating a similar problem in soil mechanics, Day [4], suggested the expressions which proved easiest to use were based on relating the mean and difference of the principal stress, to the mean and difference of the principal strains. Because the shear stiffness of typical woven fabrics is low, the material behaves essentially as an orthotropic material, so the principal stress and strain can be taken as the warp and weft stress.

Thus it appeared that a stress-strain relationship based on the following equations could be used:

$$\begin{array}{ll} \sigma_x = \text{Warp Stress} & \sigma_y = \text{Weft Stress} \\ \epsilon_x = \text{Warp Strain} & \epsilon_y = \text{Weft Strain} \\ \\ \sigma_a = (\sigma_x + \sigma_y)/2 & \tau = (\sigma_x - \sigma_y)/2 \\ \epsilon_a = (\epsilon_x + \epsilon_y)/2 & \gamma = (\epsilon_x - \epsilon_y)/2 \\ \\ \sigma_a = f(\epsilon_a) + f(\gamma) & \tau = f(\epsilon_a) + f(\gamma) \end{array}$$

For non-linear calculation various forms of function for mean stress ( $\epsilon_a$ ) and different strain ( $\gamma$ ) can be tried and in principle there is no restriction on the number of coefficients or discontinuities which may be incorporated in the coefficient of the equations. Thus in such a program each of the relationships in the equation

$$\begin{array}{ll} \sigma_a = f_1(\epsilon_a) + f_2(\gamma) & \tau = f_3(\epsilon_a) + f_4(\gamma) \\ \\ \sigma_x = \sigma_a + \tau & \sigma_y = \sigma_a - \tau \end{array}$$

were given a curve relating the stress to the strain.

### Reproduction of Test Result

To reproduce the test curve on Figure 2, values for points on the curve are derived. Consider point A on Figure 2(a), where the mean stress is 10 kN/m, and the mean strain  $\epsilon_a = 0.02$ . However, if the x and y strain were both equal to 0.02, the stress would lie on line B, i.e. there is a difference between the stresses, so it has a value for the mean strain of 0.02. From different points on the 1:1 stress ratio curve a first approximation to the curve  $\sigma_a = f(\epsilon_a)$  and  $\tau = f(\epsilon_a)$  can be made.

Data for the stresses related to ( $\gamma$ ) are obtained from the 5:1 and 1:5 curves.

Consider the curve for stress ratio (Warp : Weft) 5:1, at  $\sigma_x = 10$ ,  $\sigma_y = 2$ ,  $\epsilon_x = 0.008$  and  $\epsilon_y = 0.0055$ . Then,

$$\begin{array}{ll} \sigma_a = (\sigma_x + \sigma_y)/2 = 6 = \sigma_a' & \tau = (\sigma_x - \sigma_y)/2 = 4 \\ \epsilon_a = (\epsilon_x + \epsilon_y)/2 = 0.0031 & \gamma = (\epsilon_x - \epsilon_y)/2 = 0.0024 \end{array}$$

Now for this value of ( $\epsilon_a$ ) there will be a value of  $\sigma_a$  from  $\sigma_a = f(\epsilon_a)$ . In the final  $\sigma_a = f(\epsilon_a)$  curve,  $\sigma_a = 1.0$  for  $\gamma_a = 0.0031$ , not 6 as above. In general the value of  $\sigma_a$  will not be equal to  $\sigma_a'$  so the difference must be due to the difference strain

( $\tau$ ), i.e. data for  $\sigma_a = f(\epsilon_a)$  curve. The value of  $\tau = 4$  for  $\gamma = 0.0024$  is used as an estimate for the points on the curve can now be made using different stress levels. It is very attractive to try to use algebraic form for  $f_1, f_2, f_3$  and  $f_4$ , for example a polynomial but it was not possible because of the discontinuities in the behaviour of material. A better approach was to try multilinear curve in which each linear curve will represent the value of stress for certain range of strain.

Basically four curves required to represent the function  $f_1, f_2, f_3$  and  $f_4$ . But, it was found that four curves are not enough to reproduced all the biaxial curves required. So another two curves were added to the function  $\tau = f(\gamma)$  and  $\sigma_a = f(\gamma)$ . As a result six curves and six equations were used to defined the model parameters and to reproduced the test curves. The curves and the equations are;

Curves :

$$\sigma_a = f_1(\epsilon_a)$$

$$\tau = f_2(\epsilon_a)$$

$$\sigma_a = f_3(\gamma)$$

for  $\gamma > 0$

$$\tau = f_4(\gamma)$$

$$\sigma_a = f_5(\gamma)$$

for  $\gamma < 0$

$$\tau = f_6(\gamma)$$

Equations:

$$\sigma_x = f_1(\epsilon_a) + f_5(\gamma) + f_6(\gamma) + f_2(\epsilon_a)$$

for  $\sigma_x < \sigma_y$

$$\sigma_y = f_1(\epsilon_a) + f_5(\gamma) - f_6(\gamma) - f_2(\epsilon_a)$$

$\gamma > 0$

$$\sigma_x = f_1(\epsilon_a) + f_3(\gamma) + f_4(\gamma) - f_2(\epsilon_a)$$

for  $\sigma_x < \sigma_y$

$$\sigma_y = f_1(\epsilon_a) + f_3(\gamma) - f_4(\gamma) + f_2(\epsilon_a)$$

$\gamma < 0$

$$\sigma_x = f_1(\epsilon_a) + f_3(\gamma) + f_4(\gamma) + f_2(\epsilon_a)$$

for  $\sigma_x > \sigma_y$

$$\sigma_y = f_1(\epsilon_a) + f_3(\gamma) - f_4(\gamma) + f_2(\epsilon_a)$$

$\gamma < 0$

In the first estimate for the stress-strain curves the values for the function parameters are derived directly from the biaxial test curves. In the first estimate there will be discrepancies from the test result. Systematic adjustment of the function curves is now required, until the accuracy cannot be improved.

One way to do this is to use a program which subjects a single element, of unit size simulating a test piece. To produce the various stress strain curves, the stresses are incremented and the element strain calculated at each stress.

This program allows systematic changing of the values given for the terms of the stress strain curves. Because alteration of any one stress strain curve can alter all the resulting biaxial results, it is necessary to run all the biaxial curve for any change.

#### **IMPLEMENTATION OF FABRIC MODEL IN DYNAMIC RELAXATION ANALYSIS**

The success of numerical model in reproducing the test curve for coated fabric can be determined by implementing the model in computer program. Dynamic Relaxation Method was chosen because it can easily incorporate the nonlinear stress-strain relation and hence closely simulate the behaviour of tensile or membrane structures when subject to loading. The basis of this method is to trace step by step, for small time increments, the dynamic behaviour of a structure from the time when it is initially disturbed to the time, due to imposed viscous damping, it reaches a steady equilibrium state. The method which is proposed originally by Day[5] has been widely applied to form finding and static analysis of tension structures[6] and the solution of nonlinear problems in general.

In developing and implementing the proposed numerical model two main computer programs have been developed.

##### **Single Element Program**

This program was written to calculate the stress in a double, uniform stress triangular element based on the numerical model.

The purpose of this program is to be used as a trial program for deriving the model parameters for multi-element programs. The single element can be considered to represent any of the element of the multi-element calculation, when it is subjected to arbitrary strain or stresses, so that its behaviour can be studied in isolation. In addition, the single element can be taken as a numerical representation of specimen of fabric and made to follow numerically the imposed stress or strain of testing apparatus.

It should be emphasized that the single element program was written purely for convenience as it could be easily and quickly altered to test modifications and variations of the model parameters.

### **Multi-element Program**

The main objective of these programs was to examine convergence of the dynamic relaxation process when using the numerical model for an assembly of finite elements.

The structure chosen for this analysis is a plane fabric cruciform with slitted arms orientated along the principal directions of the fabric weave as shown in Figure 3. The analysis thus simulate biaxial testing for fabric using cruciform specimen.

Due to symmetry of loading and geometry along the principal axes, only a quadrant of the cruciform structure was analysed. The idealization employed 130 constant strain triangular elements, and along the slit double nodes and links were used to allow parting.

In trial analysis it was found that this program was difficult to converge even for a simple stress-strain relation. This may be because of the wrong idealization of the slitted arms.

To overcome this problem, the structure was simplified by discarding the slitted arms and loading was applied to the boundaries of the central square of the cruciform structure (Figure 5). Analysis for this simplified structure achieved convergence for stress ratios ranging from 0.2 to 5.

### **DISCUSSION OF RESULTS**

The model parameters obtained by the single element program for the fabric sample are summarized in Table 1 and the corresponding curves are shown in Figure 7. These model parameters were then used in the multi-element program to generate biaxial stress-strain curves for the same biaxial ratio use in the experimental result. These predicted biaxial curves were then plotted together with the experimental results, and the two sets of curves compared in Figure 2.

The model prediction for the 1 : 1 load ratio biaxial test shown in Figure 2(a). For the entire response range, the fit is good. Figure 2(b) and 2(c) show the model prediction for load ratio 5 : 1 and 1 : 5 respectively. Note that the sudden changes in the curvature and the higher initial stiffness are also reproduced.

Table 1 : Model Parameters

a1 = 2.5 as1= 0.01509	a2= 5.0 as2=0.02	a3 = 12.5 as3=0.025	a4 = 2.0 as4=0.029	a5 = 40 as5=0.037
b1=0.8 bs1=0.009	b2=1.7 bs2=0.02	b3=2.4 bs3=0.03	b4=3.0 bs4=0.04	b5=3.0 bs5=0.06
c1=1.132 cs1=0.01	c2=2.0 cs2=0.015	c3=4.5 cs3=0.025	c4=8.0 cs4=0.0277	c5=11.0 cs5=0.033
d1=2.85 ds1=0.02	d2=4.35 ds2=0.025	d3=5.0 ds3=0.0265	d4=10.0 ds4=0.03	d5=18.0 ds5=0.035
e1=2.23 es1=0.002	e2=3.52 es2=0.00275	e3=5.8 es3=0.003	e4=8.5 es4=0.0318	e5=11.0 es5=0.00467
f1=1.5 fs1=0.002	f2=2.45 fs2=0.0028	f3=4.0 fs3=0.003	f4=6.0 fs4=0.003204	f5=10.0 fs5=0.0072

Notes : a1 = Stress at point 1 (Curve a)  
as1 = Strain at point 1 (Curve a)

Note that all the theoretical curves are the best fit that can be obtained from the model. If there is an attempt to change the model parameters to get a better fit for a certain curve e.g. for 1 : 1 curve, it will change the others curves. However, as stated earlier there is always the option of adding additional terms, parameters or relationships to improve accuracy.

Another curves in Figure 8 show the prediction for fabric sample with load ratio 1 : 2 and 2 : 1. Unfortunately, because no data are available from experiment, comparison cannot be done. However, these plotted show the possibility of the model to produced biaxial response of coated fabric with any load ratio within the ranges 5 : 1 to 1 : 5.

## CONCLUSION

In this paper a numerical model representing the non-linear stress-strain properties of a coated fabric has been developed. The data or material properties required to define the model have been presented. Furthermore a systematic method of developing, testing and implementing the model was proposed and computer programs following the procedure also presented. The program was used to calibrate the model againts a sample of coated fabric. For this sample of fabric, the model predictions and experimental results for several load ratios were compared. In general, the model closely predicts the basic behaviour of coated fabrics.

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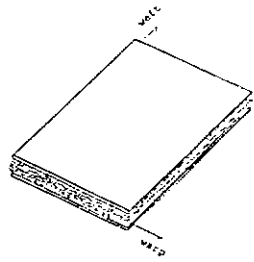


Figure 1 : Schema of Coated Fabrics

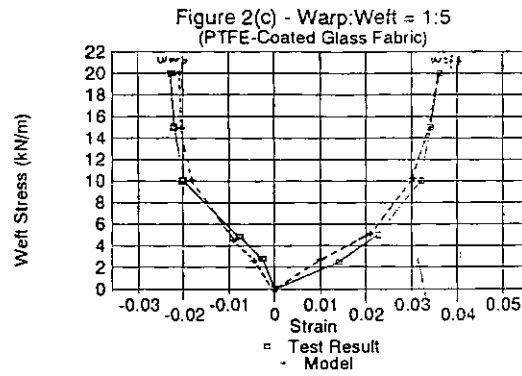
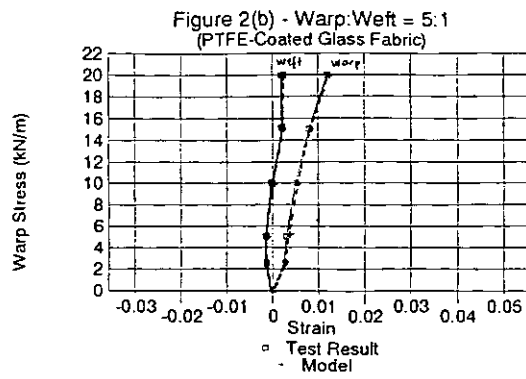
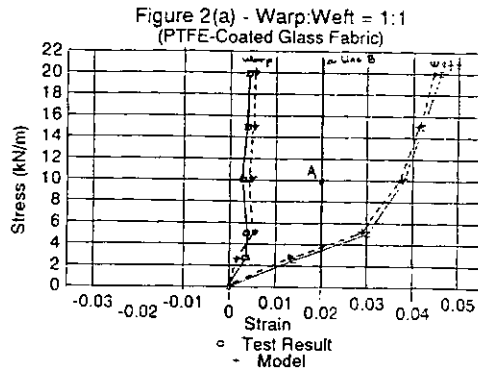


Figure 2 : Biaxial Stress-Strain Curves

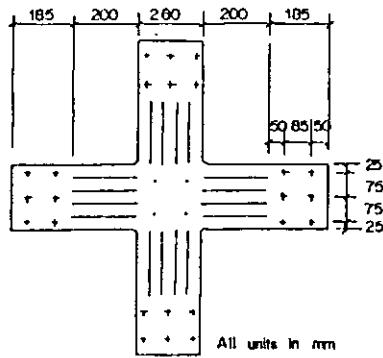


Figure 3 : Test Specimen

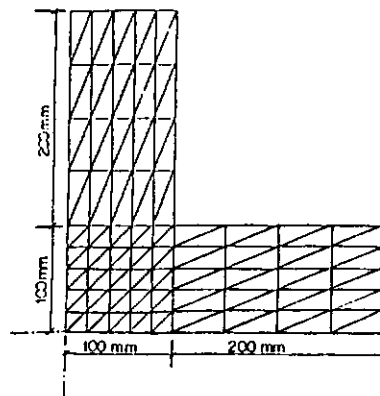


Figure 4 : Grid in one quarter of symmetrical cruciform specimen

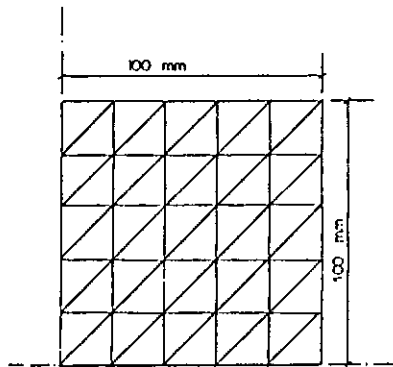


Figure 5 : Grid in one quarter of symmetrical simplified specimen for analysis

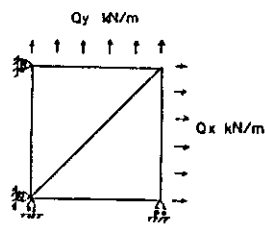


Figure 6 : Single Element

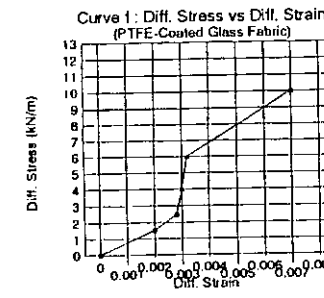
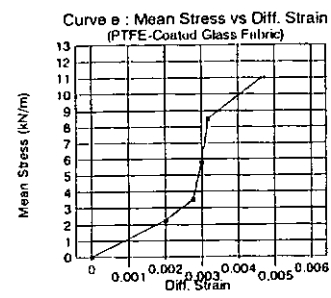
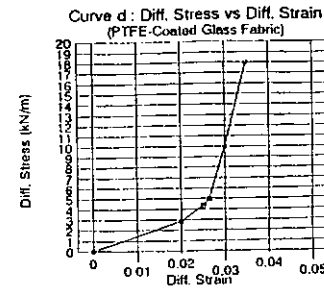
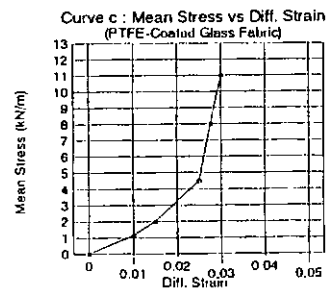
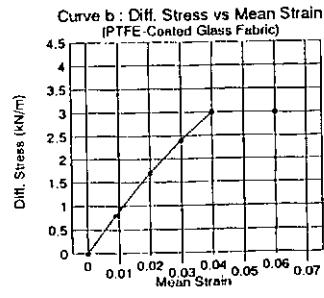
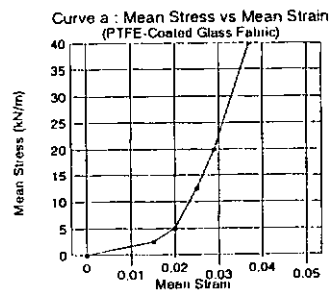


Figure 7 : Multi-linear Curves

Figure 8(a) - Warp : Weft = 2 : 1  
(PTFE-Coated Glass Fabric)

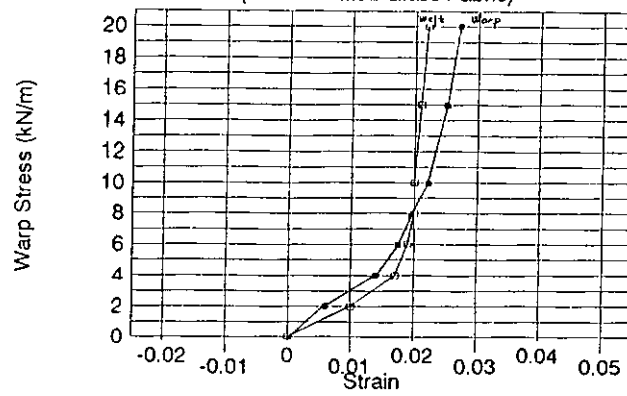


Figure 8(b) - Warp : Weft = 1 : 2  
(PTFE-Coated Glass Fabric)

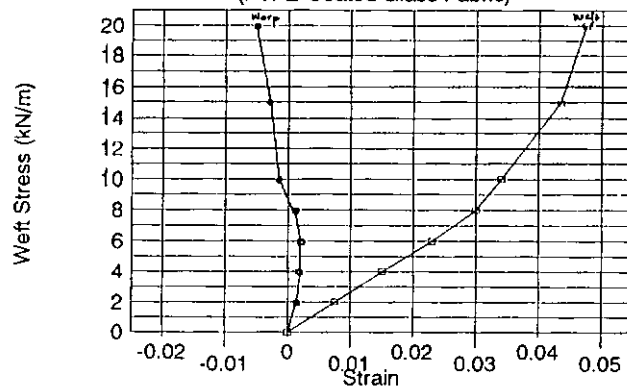


Figure 8 : Biaxial Stress-Strain Curves