Estimating Laterally Loaded Pile Response

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ABSTRACT

A simplified and practical approach for estimating the laterally loaded piles in an elastic homogeneous soil is presented. The formulation of the approach is similar to the conventional subgrade reaction theory but the response of the soil mass is evaluated by a semi-analytical solution that is related to the actual soil properties and the pile geometry instead of the conventional subgrade reaction modulus. Modified closed-form solutions for a free-head pile embedded in an elastic homogeneous soil are presented. The solutions computed by using the proposed simplified approach and the more rigorous boundary element approach for free-head piles embedded in homogeneous soils are in reasonably good agreement.

*Keywords**: Piles, Lateral Loading, Homogeneous Soil, Elastic Solutions***

INTRODUCTION

In designing pile foundations to resist lateral loads, the critical factor in some cases is the maximum pile deflection instead of the ultimate of the lateral capacity especially for long and relatively flexible piles.

The common methods of analysing the behaviour of laterally loaded piles are the subgrade reaction theory and the more rigorous continuum approaches. The subgrade reaction theory, which idealizes a pile as an elastic transversely loaded beam supported by a series of discrete linear springs representing the soil, is the most common approach used in the analysis of laterally loaded piles (Hetenyi, 1946). The main attraction of the subgrade reaction theory is its simplicity and the relatively straightforward computations, but its main disadvantage lies in the difficulty of choosing an appropriate subgrade reaction modulus for a given combination of pile size and soil type. It is usually estimated by empirical correlation that may lead in some cases to uncertainties and inaccurate solutions. The behaviour of laterally loaded piles may also be analysed by the more rigorous continuum approaches. The most versatile and powerful continuum approach is the finite element approach (Randolph 1981, Chow 1987 and Hull 1987), which is relatively more expensive and cumbersome in data preparation. The modified boundary element approach (Poulos & Davis 1980) is a more practical approach which employs the analytical point load solution of Mindlin (1936) in an elastic homogeneous half-space. The boundary integral approach (Banerjee & Davies 1978) utilizes the fundamental solution for point loads acting at the interface of two-layer elastic half-space, while Lee and Small (1991) have suggested an efficient and economical finite layer approach for piles embedded in layered isotropic and cross-anisotropic soils.

This paper presents a simplified and practical approach for estimating the laterally loaded pile response. The approach is basically similar to the conventional subgrade reaction theory, except that the response of the pile is evaluated explicitly by a semi-analytical solution, which is related to the actual soil properties and the pile geometry instead of the subgrade reaction modulus. Modified closed-form solutions for a free-head pile embedded in an elastic homogeneous soil are presented. The validity of the present approach is verified by comparing the results with solutions computed by more rigorous boundary element approach.

METHOD OF ANALYSIS

The governing equation for the deflection of a laterally loaded pile using the subgrade reaction theory is expressed as:

$$E_p I_p \frac{\partial^4 u}{\partial z^4} + k_h d\, u = 0 \qquad (1)$$

where E_p= pile Young's modulus, I_p= second moment of area of pile section, u= pile deflection, z= depth in the soil, d= diameter of pile and k_h= subgrade reaction modulus.

Equation (1) may be solved either analytically or numerically. Hetenyi (1946) has obtained closed form solutions for equation (1). There have been many empirical correlations being suggested to relate k_h to the soil modulus E_s (e.g. Skempton 1951, Terzaghi 1955, Rowe 1956, McClelland & Focht 1958, Broms 1964, Davisson 1970, Baguelin et al 1977, Pyke & Beikae 1982 and Habibagahi & Langer 1983). These correlations, which were usually derived from specific tests, depend on the pile and soil type.

The average deflection u of a pile element subjected to a uniform lateral pressure p may be expressed as follows:

$$\frac{p}{u} d = \xi E_s$$

or

$$k_h d = \xi E_s \qquad (2)$$

where

$$\xi = \frac{8\pi(1-\nu_s)}{1.13(1+\nu_s)(3-4\nu_s)\left(2\ln\frac{2L}{d} - 0.443\right)} \quad \text{(Glick, 1948)}$$

E_s= soil modulus, ν_s= soil Poisson's ratio and L= embedded pile length.

Substituting equation (2) into (1),

$$E_p I_p \frac{\partial^4 u}{\partial z^4} + \xi E_s u = 0 \qquad (3)$$

The solutions for equation (1) derived by Hetenyi (1946) may be modified by replacing ($k_h d$) with (ξE_s) for equation (3) and the solutions for a free-head pile response shown in Figure 1 are expressed as follows:

(a) Applied Head Load H

$$\text{(i)}\, u = \frac{2H\beta}{\xi E_s}\left[\frac{\sinh\beta L\cos\beta z\cosh\beta(L-z) - \sin\beta L\cosh\beta z\cos\beta(L-z)}{\sinh^2\beta L - \sin^2\beta L}\right] \quad (4)$$

$$\text{(ii)}\, \theta = \frac{2H\beta^2}{\xi E_s}\left(\frac{1}{\sinh^2\beta L - \sin^2\beta L}\right)\begin{bmatrix}\sinh\beta L(\sin\beta z\cosh\beta(L-z) \\ +\cos\beta z\sinh\beta(L-z)) \\ +\sin\beta L(\sinh\beta z\cos\beta(L-z) \\ +\cosh\beta z\sin\beta(L-z))\end{bmatrix} \quad (5)$$

$$\text{(iii)}\, M = -\frac{H}{\beta}\left[\frac{\sinh\beta L\sin\beta z\sinh\beta(L-z) - \sin\beta L\sinh\beta z\sin\beta(L-z)}{\sinh^2\beta L - \sin^2\beta L}\right] \quad (6)$$

(b) Applied Head Moment M_o

$$\text{(i)}\, u = \frac{2M_o\beta^2}{\xi E_s}\left(\frac{1}{\sinh^2\beta L - \sin^2\beta L}\right)\begin{bmatrix}\sinh\beta L(\cosh\beta(L-z)\sin\beta z \\ -\sinh\beta(L-z)\cos\beta z) \\ +\sin\beta L(\sinh\beta z\cos\beta(L-z) \\ -\cosh\beta z\sin\beta(z-L))\end{bmatrix} \quad (7)$$

$$\text{(ii)}\, \theta = \frac{4M_o\beta^3}{\xi E_s}\left[\frac{\sinh\beta L\cosh\beta(L-z)\cos\beta z + \sin\beta L\cosh\beta z\cos\beta(L-z)}{\sinh^2\beta L - \sin^2\beta L}\right] \quad (8)$$

$$\text{(iii)}\ M = \frac{M_o}{\sinh^2 \beta L - \sin^2 \beta L} \begin{bmatrix} \sinh \beta L \big(\sinh \beta (L-z) \cos \beta z + \cosh \beta (L-z) \sin \beta z \big) \\ - \sin \beta L \big(\sinh \beta z \cos \beta (L-z) + \cosh \beta z \sin \beta (L-z) \big) \end{bmatrix} \quad (9)$$

where $\beta = \left(\dfrac{\xi E_s}{4 E_p I_p} \right)^{1/4}$, u=pile deflection, θ= pile rotation and M=pile moment.

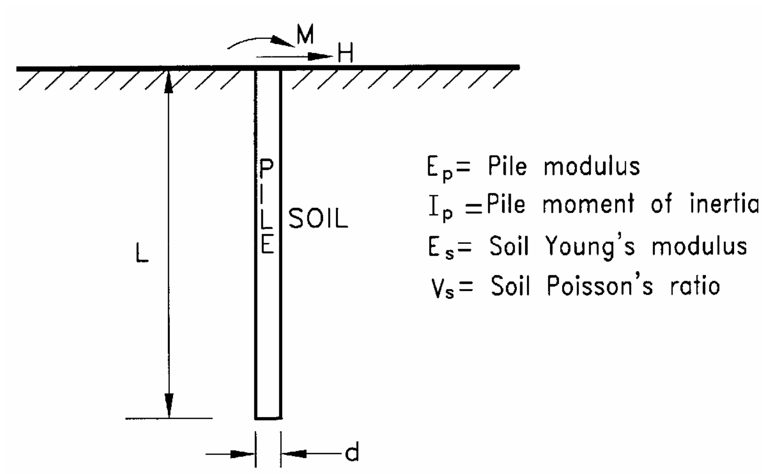

Figure 1 : Definition of Laterally Loaded Pile Problem

COMPARISON WITH MORE RIGOROUS BOUNDARY ELEMENT SOLUTIONS

The validation of the present approach is assessed by comparison with the linear elastic solutions from the more rigorous boundary element approach BE (Poulos & Davis 1980) for free-head piles embedded in an elastic homogeneous soil.

Figure 2 shows the effect of pile flexibility factor ($K_R=E_pI_p/E_sL^4$) and pile slenderness ratio (L/d) on the normalized pile head deflection (u_oE_sL/H). The present solutions are comparable with those computed by the boundary element approach except for very flexible piles in which the present approach computed higher pile head deflections.

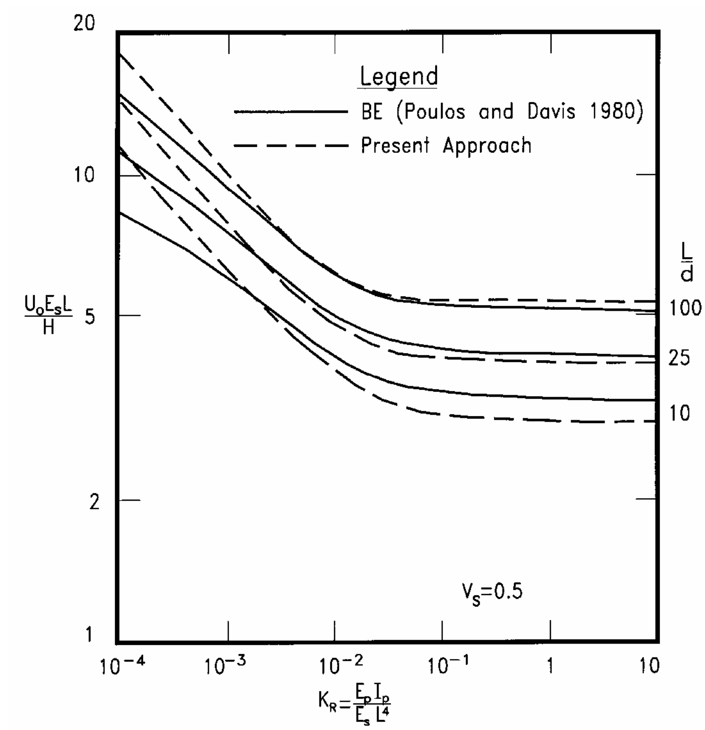

Figure 2 : Normalized Head Deflection Response for Free-Head Pile

The influence on normalized head rotation with respect to moment (which is also equivalent to head rotation θ_o normalized with respect to load) of the pile flexibility factor and pile slenderness ratio is shown in Figure 3. The solutions computed by both approaches are in close agreement except for very flexible piles.

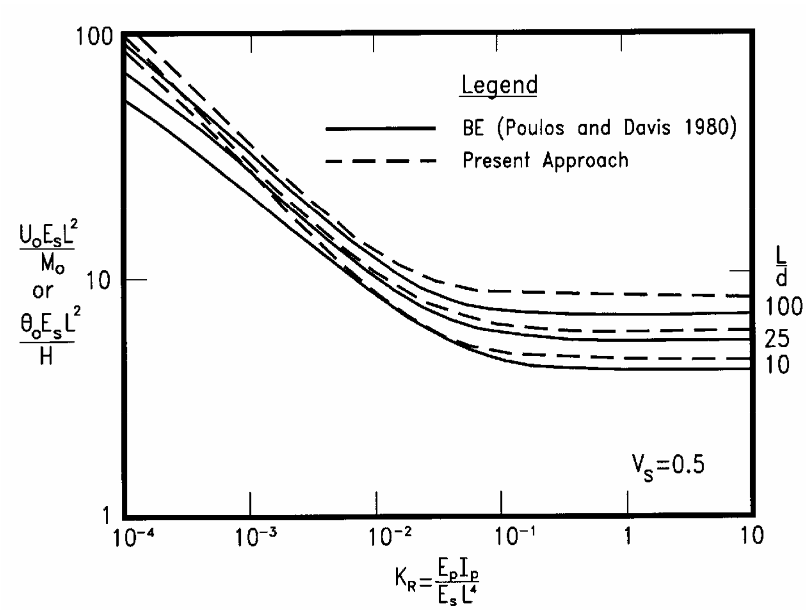

Figure 3 : Normalized Head Deflection (or Rotation) Response for Free-Head Pile

Figure 4 illustrates the effect on the normalized head rotation with respect to a moment M_o of the pile flexibility factor and pile slenderness ratio. The solutions computed by both approaches are similar.

Figure 5 shows the effect of pile slenderness ratio on typical pile deflection profile along a free-head pile. The present approach computed slightly lower pile deflections than those computed by the boundary element approach.

The variation of moment distribution along a pile with pile flexibility factor and slenderness ratio is depicted in Figure 6. For horizontal head loading only, the maximum moment typically occurs at a depth of between 0.1L and 0.4L below the ground surface, the greater depth occurring for stiffer piles. For moment loading only, the maximum moment always occur at the ground surface and equals to the applied moment. The moment distributions along a pile computed by the present approach and boundary element approach are in good agreement.

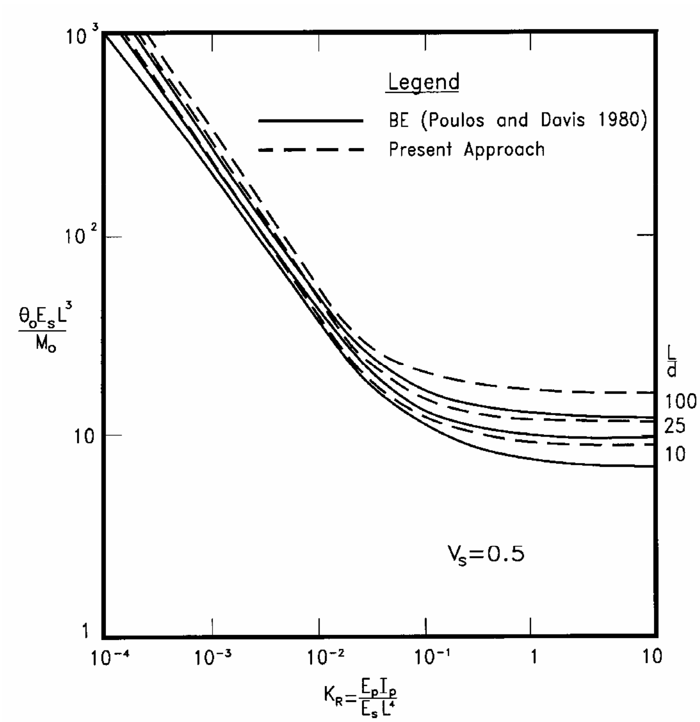

Figure 4 : Normalized Head Rotation Response for Free-Head Pile

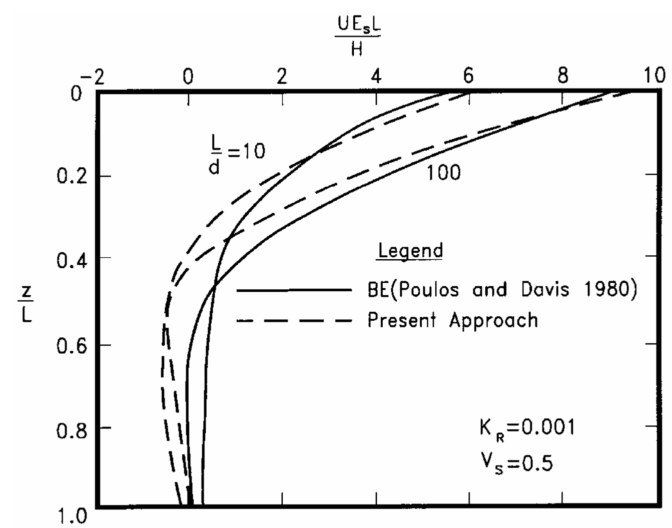

Figure 5 : Deflection Profile Along Free-Head Pile

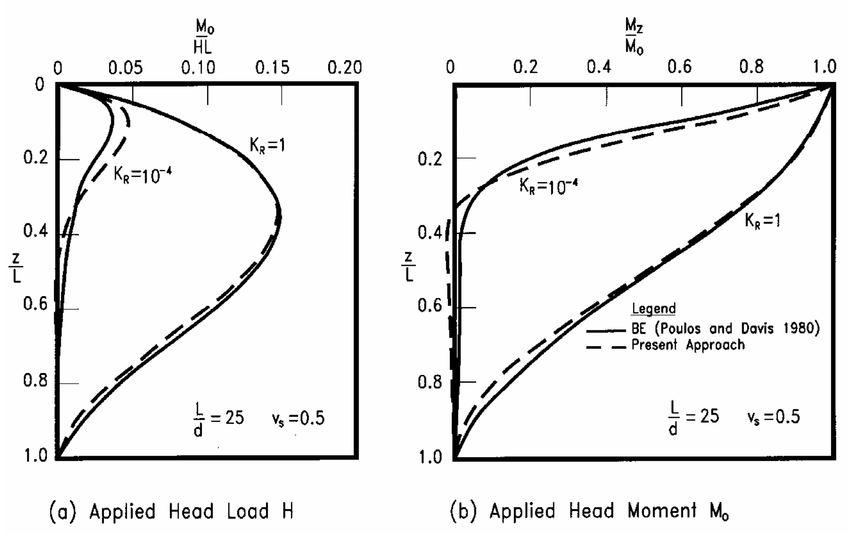

Figure 6 : Moment Profile Along Free-Head Pile

The deflection and rotation for very flexible or slender piles computed by the boundary element approach are generally lower than those computed by the present approach. Because of the limited number of elements used in the boundary element approach, the solutions may be somewhat inaccurate for piles that are very slender or flexible, and may lead to underestimates of deflection and rotation (Poulos &Davis 1980).

CONCLUSIONS

A simplified and practical approach for estimating the response of laterally loaded piles embedded in an elastic homogeneous soil is presented. Although the formulation of the present approach is basically similar to the conventional subgrade reaction theory the response of the soil is evaluated by a semi-analytical solution, which is related to the actual soil properties and the pile geometry instead of the conventional subgrade reaction modulus. Modified closed-form solutions for a free-head pile embedded in an elastic homogeneous soil are presented. Despite some differences in magnitude between the present approach and the more rigorous boundary element approach for very slender or flexible piles, the overall results are in reasonably good agreement.

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