# FLOOD FORECASTING USING FGM MODEL IN CHINDWIN RIVER BASIN

Win Win Zin<sup>1\*</sup>, F. Nestmann<sup>2</sup>, J. Ihringer<sup>2</sup>

<sup>1</sup>Department of Civil Engineering, Yangon Technological University, Myanmar <sup>2</sup> Institute of Water and River Basin Management, Karlsruhe University, Germany

\*Corresponding Author: winzin@mptmail.net.mm

Abstract: The present research work deals with flood forecasting in Chindwin River basin which is situated in Northern West of Myanmar under available hydro-meteorological data. Flood is one of the natural disasters which occur in Myanmar every year. Flood forecasting and issues of flood warnings are the effective ways to reduce damages. The goal of the study has been to initiate operational flood forecasting. This research applied Flussgebietsmodell (FGM) which is originally developed by the Institute of Hydrology and Water Resources Planning (IHW) of the University of Karlsruhe, Germany. FGM model is an event-based rainfall-runoff model. Model parameters are runoff coefficient, unit hydrograph parameters and routing parameters. Unit hydrograph is determined using linear cascade model. The effective precipitation is routed to the outlet through a linear transfer function that is assumed to be time invariant. Flood from each subbasin is calculated by means of effective rainfall convoluted with a unit hydrograph. Flood routing is done by Kalinin - Milyukov method. The two important parameters, when predicting a flood hydrograph, are the time to peak discharge and the magnitude of the peak discharge. It was found that the FGM model has been able to predict this information with acceptable accuracy. The model performs quite well especially for the floods where relation between rainfall and runoff is good. The numerical verification criteria used in model calibration are Nash efficiency and coefficient of variation of the residual of error. Model efficiencies obtained in calibration periods are good efficiency. In this study, it is seen that the diagnosis performs well. Therefore, the FGM model is generally considered to be suitable for flood forecasting in Myanmar catchments.

Keywords: runoff coefficient; unit hydrograph; Kalinin, Nash efficiency.

## 1.0 Introduction

Flood is one of the natural disasters which occur in Myanmar almost every year. Among the four main rivers of Myanmar, the occurrence of major floods in the rivers of Ayeyarwaddy and Chindwin are mostly associated with the pronounced monsoon. The present condition of the Chindwin river basin is featured by its abundance of river water, large difference in rainfall, runoff and water level in a year, swift currents and whirlpools in the rainy season and chronic flood damages in the rainy season. Accurate flood forecasting and issues of flood warnings are the effective ways to reduce damages.

The location of study area is shown in Fig. 1. The Chindwin river has the catchment area of 115,300 km<sup>2</sup>. Length of the river is approximately 1046 km. Wide and thickly developed river terraces are frequently seen on the both banks along the river. There are only few gauged stations within the catchment area. In the scale of the investigated river basin, it has to be emphasised that the spatial pattern of rainfall intensities is essential. Real-time forecasts will be very dependent on the broad availability and accuracy of input data. Fifteen years of daily rainfall data at Putao, Hkamti, Homalin, Mawlaik, Kalaewa and Monywa and fifteen years of daily discharge data at Hkamti, Homalin, Mawlaik and Monywa are used in this study. Fig. 2 shows the catchment of Chindwin river.

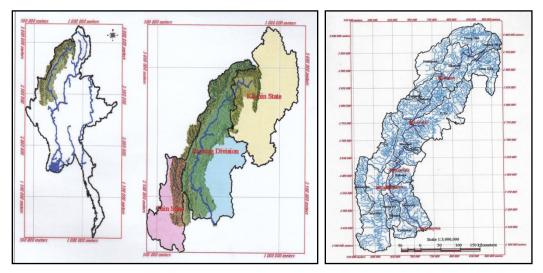


Figure 1: Location of study area

Figure 2: Chindwin river basin

### 2.0 Rainfall – Runoff Modelling for Flood Forecasting

Hydrological models of various types have been extensively used for water resources planning and management. Types of models can be classified as empirical models, regression models, rainfall-runoff models, flood routing models and data driven models. Rainfall-runoff models which are mostly used for inflow and flood forecasting for operational use. These models are easier to set up and need less data requirement. Difficulty of flood forecasting is mainly due to the complication of processes through which rain water takes its several paths to reach a river channel. At the same time, it is important to recognize that the rainfall runoff process is inherently spatial, nonlinear and time-variant, whereas many models are lumped linear and time-invariant. Real-time forecasts will be dependent on the accuracy of input data, particularly of spatial patterns of rainfall intensities.

Modelling of an event based rainfall-runoff process has been of importance in hydrology. Historically researchers have relied on conventional modelling techniques, either deterministic, which consider the physics of the underlying process, or systems theoretic/black box, which do not. Deterministic models of varying degrees of complexity have been employed in the past for modelling the rainfall-runoff process with varying degrees of success. The rainfall-runoff process is a complex, dynamic, and nonlinear process, which is affected by many factors, and often interrelated, physical factors (Ashu Jain and Prasad Indurthy 2003). There are various event based rainfall-runoff modelling techniques such as deterministic, statistical and artificial neural networks. In this study, the deterministic unit hydrograph method is applied.

FGM model was tested in the study area. FGM model was originally developed by Institute of Hydrology and Water Resources Planning (IHW), University of Karlsruhe. The FGM is named after the abbreviation of 'Flussgebietsmodell'. This software package is available free for research students. A rainfall-runoff model can be one of two types- an event based rainfall-runoff model or continuous rainfall-runoff model. FGM model is an event based rainfall-runoff model. The model has semi-distributed spatial structure to be applied on homogeneous units of a catchment. However, the model may also be applied in a lumped mode. For calibration of such model, several rainfall-runoff events are needed. The model approach has proved flexible and robust in solving water resource problems. This model has been applied in several catchments of Germany. It is so versatile and efficient that it can be used by any engineer with a hydrology background.

#### 3.0 FGM Model Structure

The model is based on the unit hydrograph as fundamental building element. The basin is subdivided into a system with node points. Node points are placed at the locations of gages, retention basins, river junctions, and critical points for which flood discharges are required. For each point, the model calculates and stores the flood wave as a function of time, with time intervals ranging from a few minutes to the daily time step, depending on the catchment size. The schematic diagram

of FGM model is shown in Fig. 3. The model requires subdividing a catchment into small sub catchments which can be connected to form a basin model. The flood from each sub basin is calculated by means of a design rainfall pattern convoluted with a unit hydrograph. Flood routing models are used to connect the sub catchments.

Between entrance and outlet three options are available. The first option yields the inflow from the lateral area, which is obtained by means of a difference in regionalized unit hydrograph for rural areas. The second option consists of the inflow from an urbanized area, which can be represented either by a simple urban area model, or by a more elaborate model which permits to simulate the structure of the storm drainage system, as is shown in the upper part of Fig.3. The inflow (upper left corner) from other urban areas not directly connected to the river, mixes in a main urban sewer with two inputs obtained from rainfall: an urban part and a rural part, each with its separate transfer function. Retention both in the sewer system itself as well as in retention basins for pollutants and water can be considered. Discharge limitations by valves are sometimes required, and also diversions of sewage into sewage treatment plants, so that a highly controlled amount of discharge from an urban area enters into the river. The third option is that of a flood storage reservoir, whose characteristic functions (such as the stage storage curve, the hydraulic functions describing the spillway and outlet characteristics) are stored as functions. River reach model is available as subroutine, by means of which the flood wave at the inlet is routed to the outlet. Model has different modules and this modular structure is flexible to use.

The basis of the flood determination is the unit hydrograph, coupled with models of unsteady river flow. The regionalized unit hydrograph can be developed empirically. Two basic types of region are distinguished: urban regions, and rural areas. The output of the rainfall-runoff model is mostly dependent on the amount of rainfall which falls on the catchment. The design storm is characterized by its duration for which depth-duration diagrams with recurrence interval T can be used if it has been prepared. Intensity-duration-frequency curves are useful in urban storm-drainage design and other applications. Rainfall distribution can be considered in space and time. Space distribution of the extreme rainfall depth over the duration can be found from the curve which has been obtained by trial and error with many different distributions. Model itself has complexity to some extent. Model performance depends on the available data.

In FGM model, runoff generation is done by runoff coefficient approach. Runoff concentration within the drainage basin approximately is represented by a linear system. The lateral runoff flow from the model elements to the channel segments is considered as runoff concentration. Runoff concentration depends on catchment area and catchment properties such as slope, soil cover, subsurface conditions and river network. Model parameters are runoff coefficient, unit hydrograph parameters and routing parameters. Number of model parameters depends on the method applied.

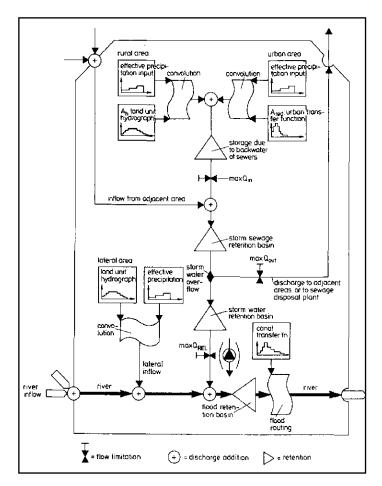


Figure 3. Schematic diagram of FGM model Source: Plate, Ihringer and Lutz (1988)

# 4.0 Analysis and Simulation of Hydrological Processes using FGM Modules

A purpose of hydrograph analysis is to analyze measured rainfall and runoff data to obtain an estimate of the transfer function. Once the transfer function has been developed, it can be used with design storm to compute the runoff that would be expected. In the analysis phase, the hyetograph and hydrograph are known and the transfer function is estimated. In the synthesis phase, a rainfall hyetograph and a synthetic transfer function are used to compute the runoff hydrograph (McCuen 1989).

In hydrograph analysis phase, unit hydrograph derivation is done by both non parametric approach and parametric approach. And it is noticed that unit hydrographs derived from non parametric approach have more variation than from parametric approach (such as conceptual models). The unit hydrographs model structure determined from parametric approach is suitable to represent watershed behaviour. It can provide more important information about catchment characteristics. Therefore, unit hydrographs derived from parametric approach are applied in this study.

Unit hydrograph parameters are determined by using linear reservoir cascade method in the study. Nash (1957) proposed a cascade of linear reservoirs of equal sizes to represent the instantaneous unit hydrograph for a catchment. All the reservoirs have the same storage constant k. Number of identical reservoirs required to the model is computed from an observed event of direct runoff hydrograph and the corresponding effective rainfall hyetograph. The model is lumped and time invariant. The routed outflow from the first reservoir becomes the input to the second reservoir in series and the second reservoir output becomes the input to the third, and so on. Output from the last (n<sup>th</sup>) reservoir is the output from the system representing an instantaneous unit hydrograph for the basin (Patra 2003).

The unit hydrograph of the Nash model is:

$$u(t) = \frac{1}{k\Gamma(n)} \left(\frac{t}{k}\right)^{n-1} e^{-t/k}$$

where  $\Gamma(n) = Gamma \text{ function}$  n = Number of linear reservoirsk = Storage constant **(** )

Parameters n and k for Nash model can be determined by method of moment, maximum likelihood method, method of least squares, and principle of maximum entropy. In this study, parameters n and k are determined by method of moment.

In FGM model, simple base flow separation is applied. For the present case study, it is seen that Chindwin river has high base flow. For this, base flow separation is not done by simplified method. In this study, base flow separation is done by Rodriguez method. The method proposed by Rodriguez (1989) to represent the baseflow component is

$$QB_{i}^{L} = \frac{a}{1+b}QB_{i-1}^{L} + \frac{b}{1+b} (Q_{i}^{L} - QB_{i-1}^{L})$$
(2)

$$QR_i^L = Q_i^L - QB_i^L \tag{3}$$

where  $QR_{i}^{L}$ ,  $Q_{i}^{L}$ , and  $QB_{i}^{L}$  represent respectively rapid storm flow, total runoff and baseflow for i<sup>th</sup> time step of event L. This method involves two parameters *a* and *b*, which can be related to the coefficient of the exponential recession.

If it is assumed that beyond a given time step, the rapid runoff is over then

$$Q_i^L = Q B_i^L \tag{4}$$

which, combined with Eq. (2), gives

$$QB_i^L = (a-b)QB_{i-1}^L \tag{5}$$

where (a - b) can be obtained from recession analysis. Usually the parameter value is optimized by trial and error (Duband et al 1993).

In FGM model, different methods available for estimating effective rainfall are constant fraction, initial loss and constant loss rate, exponential method and  $\phi$  index method. Initial loss value can be given if we consider it in the process. For the determination of the runoff coefficient, different methods are used depending on the availability of data. SCS method, coaxial diagram method and Lutz method are also available for runoff coefficient determination in FGM model. Choice and validity of rainfall loss methods depend on the type of problem, the data available and the runoff processes which are likely to be dominant (Maidment 1993).

In FGM model, different methods available for flood routing are:

- (1) Muskingum method;
- (2) Kalinin-Milyukov method;
- (3) Kalinin-Milyukov with wave forming;
- (4) Linear reservoir cascade;
- (5) Double reservoir cascade;
- (6) Translation and
- (7) Translation-Diffusions.

In this study, the Kalinin-Milyukov method is used. The Kalinin-Milyukov method was developed by Kalinin-Milyukov (1957) and involves successive routing through a characteristic reach. Its operation is equivalent to that of a cascade of n equal linear reservoirs, each of delay time k (Singh 1988). The Kalinin-Milyukov method is based on the principle of the reservoir cascade. The application of this method requires some input about the geometry of the river cross sections.

Stationary outflow can be determined as follows:

$$Q_{st} = k_{st} A R^{\frac{2}{3}} I_{st}^{\frac{1}{2}}$$
(6)

where  $Q_{st} = Stationary outflow (m^3/sec)$   $k_{st} = Stickler's roughness coefficient (m^{1/3}/sec)$   $A = Cross sectional area of channel (m^2)$  R = Hydraulic radius (m) $I_{st} = Energy line gradient for stationary flow$ 

A channel cross section with characteristic length  $L_c$  may be regarded as a reservoir. The characteristic length is determined as follows:

$$L_c = \frac{Q_{st}}{I_{st}} \frac{d(h_{st})}{d(Q_{st})}$$
(7)

where

 $L_c$  = Characteristic length

 $Q_{st} = Stationary outflow$ 

 $h_{st} = Water depth$ 

 $I_{st}$  = Energy line gradient for stationary flow

Mean characteristic length is obtained as

$$L_{cm} = \frac{1}{m} \sum_{i=1}^{m} L_{c,i}$$
(8)
$$L_{cm} = Mean \text{ characteristic length}$$

where

 $L_{c,i}$  = Characteristic length

m = Number of intervals in outflow curve

Number of reservoirs is determined from

$$N = \frac{L}{L_{cm}} \tag{9}$$

Where N = Number of reservoirs L = Total length of river reach  $L_{cm} =$  Mean characteristic length

The storage constant 'K' can be determined by the use of stationary outflow curve and the cross-sectional shape.

$$K = I_c \cdot b \cdot \frac{dh}{dQ_{st}} \tag{10}$$

where

K = Storage constant

 $I_c$  = Characteristic length b = Width of the channel

 $Q_{st} = Stationary outflow$ 

dh = Differential water depth

There are altogether five model parameters to be found out in the study. They are runoff coefficient (C), two unit hydrograph parameters (n, k) and two routing parameters (N, K).

#### 5.0 Model Application and Results

In this study, rural runoff with flood routing (first option) is done. Rainfall distribution is considered as uniform distribution according to available data. Fifteen year data of daily rainfall and runoff are taken for calibration as well as verification. River basin is subdivided into four sub-catchments namely, Hkamti,

Homalin, Mawlaik and Monywa. Subdivision of basin is simply done based on gauging station nodes because of limited data availability. First of all, overall hydrographs are separated to components by flood routing using Nash method. Next, hydrographs for subcatchments are assigned by differencing observed discharge values and routed discharge values. Base flow separation is done by Rodriguez method. In base flow separation, recession constant is determined based on good recession curves. Then parameters 'a' and 'b' are optimized by trial and error.

Several flood events are analyzed in each subcatchment. Flood events are classified based on dominating rainfall patterns. For example, clear flood events are chosen in Homalin subcatchment. Hkamti rainfall dominated events and Homalin rainfall dominated events are differentiated. It is seen that upper gauge station's rainfall is more frequently dominated than lower gauge station's rainfall. Average unit hydrograph parameters and average runoff coefficients are determined based on several clear single storms. Table 1 shows average runoff coefficients and average unit hydrograph parameters. Using average parameters, direct runoff hydrographs are simulated for single events. Finally flood hydrographs are simulated for flood season (multiple events) and optimum parameters are determined. Table 2 shows optimum runoff coefficients and optimum unit hydrograph parameters.

In FGM model, flood routing parameters can be roughly estimated using Kalinin-Milyukov method. For estimating these parameters, necessary input data are river bed profile, slope of river, length of river reach and stickler coefficient. By adjusting these rough estimated routing parameters, optimum flood routing parameters are determined. It is seen that routing parameters are not so sensitive in model calibration. Optimum flood routing parameters are shown in Table 3.

Station	С	n	k
			(hour)
Hkamti (Hkamti rainfall dominated) (Putao rainfall dominated)	0.40 0.25	1.67 2.24	40.0 24.6
Homalin (Hkamti rainfall dominated) (Homalin rainfall dominated)	0.30 0.32	1.91 1.55	26.5 31.2
Mawlaik (Homalin rainfall dominated) (Mawlaik rainfall dominated)	0.24 0.28	2.14 1.93	33.8 35.2
Monywa (Mawlaik rainfall dominated) (Kalaewa rainfall dominated)	0.16 0.22	3.82 3.12	29.6 27.7

Table 1: Average runoff coefficient (C) and average unit hydrograph parameters (n, k)

Table 2: Optimum runoff coefficient (C) and optimum unit hydrograph parameters (n, k)

Station	С	n	k (hour)	
Hkamti	0.40	2.0	40.0	
Homalin	0.26	2.0	30.0	
Mawlaik	0.20	2.5	28.0	
Monywa	0.14	3.0	26.0	

Table 3: Flood routing parameters (N, K) using Kalinin-Milyukov method

Catchment	Number of Reservoirs	Storage Constant (hour)	
Hkamti-Homalin	25	3.0	
Homalin-Mawlaik	25	1.5	
Mawlaik-Monywa	30	1.1	

The comparison of observed and simulated discharges for severe flooded years is shown in Fig. 4 to Fig. 15. In the plotted diagrams, horizontal axis and vertical axis describe time (hr) and discharge  $(m^3/s)$  respectively. Monsoon season is normally from middle of May to end of October. Sometimes monsoon season starts in June.

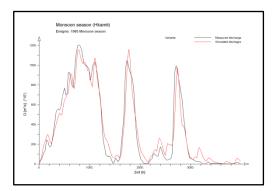


Figure 4: Comparison of measured and simulated discharge at Hkamti (1995)

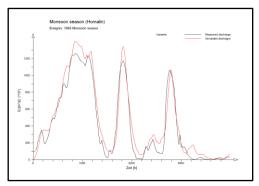


Figure 5: Comparison of measured and simulated discharge at Homalin (1995)

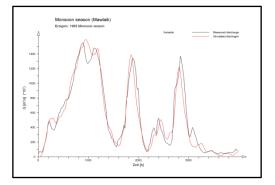


Figure 6: Comparison of measured and simulated discharge at Mawlaik (1995)

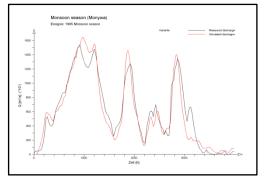


Figure 7: Comparison of measured and simulated discharge at Monywa (1995)

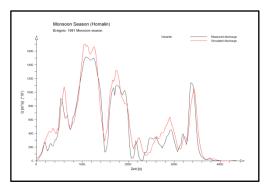


Figure 8: Comparison of measured and simulated discharge at Hkamti (1991)

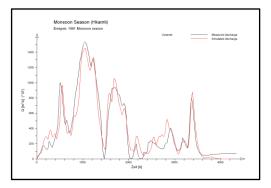


Figure 9: Comparison of measured and simulated discharge at Homalin (1991)

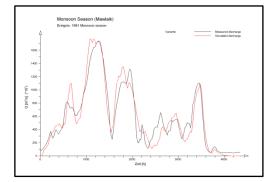


Figure 10: Comparison of measured and simulated discharge at Mawlaik (1991)

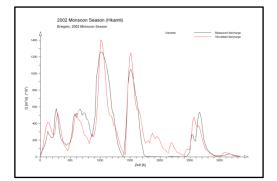


Figure 12: Comparison of measured and simulated discharge at Hkamti (2002)

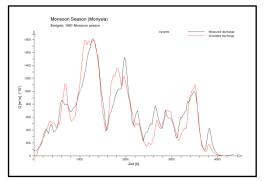


Figure 11: Comparison of measured and simulated discharge at Monywa (1991)

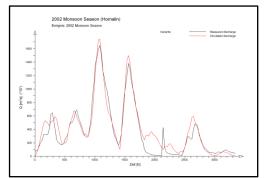


Figure 13: Comparison of measured and simulated discharge at Homalin (2002)

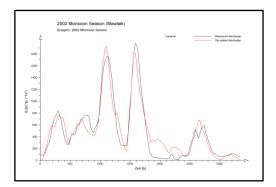


Figure 14: Comparison of measured and simulated discharge at Mawlaik (2002)

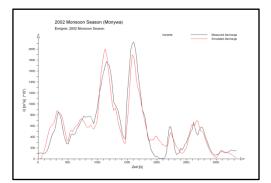


Figure 15: Comparison of measured and simulated discharge at Monywa (2002)

#### 5.1 Model Performance

The model efficiency (EFC) and coefficient of variation of the residual of error (CVE) are considered as the standard indices for evaluation of the agreement between observed and calculated discharge in this study. Nash efficiency ( $\mathbb{R}^2$ ) is computed from the following equation.

$$R^{2} = \frac{\sum (Q_{o} - \bar{Q}_{o})^{2} - \sum (Q_{c} - \bar{Q}_{o})^{2}}{\sum (Q_{o} - \bar{Q}_{o})^{2}}$$
(11)

The higher the model efficiency value, the better the model fit. Coefficient of variation of the residual of error is computed from following equation.

$$CVE = \frac{\left[\frac{\sum(Q_c - Q_o)^2}{n}\right]^{\frac{1}{2}}}{\bar{Q}_o}$$
(12)

where

CVE = Coefficient of variation of the residual of error

 $Q_c = Computed runoff$ 

 $Q_o = Observed runoff$ 

Model efficiency and coefficient of variation of the residual of error for severe flooded years (1991, 1995, 2002 and 2003 monsoon season) are shown from Table 4 to Table 7. It is seen that 1991 and 2002 are the worst flooded years among the records.

Year	EFC	CVE
1991	0.907	0.314
1995	0.951	0.252
2002	0.824	0.507
2003	0.932	0.274

Table 4: Model efficiency and coefficient of variation of the residual of error at Hkamti

Table 5: Model efficiency and coefficient of variation of the residual of error at Homalin

Year	EFC	CVE
1991	0.898	0.305
1995	0.943	0.252
2002	0.841	0.403
2003	0.934	0.206
2003	01221	
	coefficient of variation of the rest	idual of error at Mawlaik
		idual of error at Mawlaik
able 6: Model efficiency and	coefficient of variation of the rest	
able 6: Model efficiency and Year	coefficient of variation of the res	CVE
able 6: Model efficiency and Year 1991	coefficient of variation of the rest         EFC         0.897	CVE 0.253

Table 7: Model efficiency and coefficient of variation of the residual of error at Monywa

Year	EFC	CVE
1991	0.841	0.274
1995	0.908	0.234
2002	0.887	0.302
2003	0.872	0.246

It is noticed that measured discharge values of 2003 at Mawlaik and Monywa are not reliable. Some measured discharge values at Hkamti and Homalin in 2003 are higher than measured discharge values at Mawlaik and Monywa. It is seen that measured discharge values of 2002 (after the flood event) at Hkamti and Homalin are not reasonable. It may be due to rating curve or data error. Rating curves for these stations should be upgraded.

### 6.0 Conclusion

The two important parameters, when predicting a flood hydrograph, are the time to peak discharge and the magnitude of the peak discharge. It was found that the FGM model have been able to predict this information with acceptable accuracy. This shows that the FGM model can be efficient in modelling an event-based rainfall-runoff process for determining peak discharge and time to the peak discharge accurately. It is noticed that peak error and difference between summation of observed and simulated discharge volume are quite satisfactory.

It is better to use a simple model with acceptable accuracy, rather than a complex model without the required high data density which is commonly not available in the investigated large scale and in the practice of water resources management. A complex model might produce a better fit in model calibration, but it is unstable under changeable conditions. In this context, FGM has an appropriate degree of complexity. It is noticed that especially sensitive in base flow separation. It is seen that it could not beautifully pick up the base flow throughout the season if simple base flow separation is done. Therefore, base flow separation is done by Rodriguez method in the study. The setting of the base point of recession on the falling limb of the hydrograph greatly affects the amount of surface runoff calculated. The model suffers from uncertainty due to difficulties in estimating the loss function parameters which are prone to large variations but the losses may not be significant for extreme floods. Assumption of uniformly distributed rainfall intensities within the catchment reduces model efficiency. Having less rainfall stations makes difficulty in analysis of flood events in the study.

It is seen that peak error is not more than ten percent in some flood events. The model performs quite well especially for the floods where relation between rainfall and runoff is good. Thus, it depends on the quality of input data. There are only few rain gauged stations in the study area. Therefore more rain gauged stations are necessary to be installed in the investigated river basin. The numerical verification criteria used in model calibration are model efficiency and coefficient of variation of the residual of error. In this study, it is seen that the diagnosis performs well. Therefore, the FGM model is generally considered to be suitable for flood forecasting in Myanmar catchments.

#### Acknowledgement

Two anonymous reviewers are thanked for their helpful and constructive criticism.

### **References:**

- Ashu Jain and Prasad Indurthy. (2003). Comparative Analysis of Event–based Rainfall Runoff Modelling Techniques-Deterministic, Statistical, and Artificial Neural Networks, Journal of Hydrologic Engineering, 8(2): 93-98.
- Duband, D., Obled, Ch., and Rodriguez, J.Y. (1993). Unit Hydrograph Revisited: An Alternate Iterative Approach to UH and Effective Precipitation Identification, Journal of Hydrology. 150: 115-149.
- Kalinin, G.P., and Miyukov, P.I. (1957). On the Computation of Unsteady Flow in Open Channels, Meteorologia Gidrologiya Zhuzurnal Leningrad, USSR, 10: 10-18.
- Maidment, D.R. (1993). Handbook of Hydrology. New York, McGraw Hill Inc.
- McCuen, R.H. (1989). Hydrologic Analysis and Design, Prentice Hall, Englewood, Cliffs, New Jersey.
- Nash, J.E. (1957). The Form of the Instantaneous Unit Hydrograph, Hydrol. Sci. Bull. 3: 114-121.
- Patra, K.C. (2003) .Hydrology and Water Resources Engineering, Alpha Science International Ltd, CRC Press LLC.
- Plate, E.J., Ihringer, J. and Lutz, W. (1988). Operational Models for Flood Calculations, Journal of Hydrology (100) 489-506.
- Rodriguez, J.Y.(1989) Nouvelles Perspectives de Developpement dans la Modelisation des Pluies Efficaces par Application de la Methode DPFT. IAHS Third Scientific Assembly-Symposium on New Directions for Surface Water Modelling. Baltimore, MD, May 1989. IAHS Publ., 181:235-244. Cited in Duband, D., Obled, Ch., and Rodriguez, J.Y. 1993. "Unit Hydrograph Revisited: An Alternate Iterative Approach to UH and Effective Precipitation Identification" Journal of Hydrology 150: 115-149.
- Singh, V.P. (1988). Hydrologic Systems Volume I –Rainfall-Runoff Modelling, Englewood Cliffs, N. J: Prentice-Hall.