Malaysian Journal of Civil Engineering 25(1):82-93 (2013)

DYNAMIC ANALYSIS OF COMPLEX STRUCTURE BY EQUIVALENT METHODS: APPLICATIONS TO ISOTROPIC AND ORTHOTROPIC PLATES

Abdelhak Sekhri[1]*, Toufik Benmansour[2], Abd Elyamine Mebarki[3] & Samy Hamada[2]

[1]*Hadj Lakhdar University, 05000 Batna, Algeria.*
[2]*Mentouri University, 25000 Constantine, Algeria.*
[3]*Kasdi Merbah University, 30000 Ouargla, Algeria*

**Corresponding Author*: abdelhaksekhri@gmail.com

Abstract: This paper intends to provide an equivalent method for the evaluation of natural frequency of isotropic and orthotropic thin rectangular plates with different restraint conditions. Starting from a simple and a general approximate formula for the frequency, which is the extension of Hearmon's expressions presented for the fundamental mode; it is shown how to calculate the fundamental mode of isotropic and orthotropic rectangular plates using the proper coefficient values already available in the scientific literature. For the higher modal frequencies, a particular form of Rayleigh's method is proposed, leading to a simple procedure to calculate the fundamental frequency. In fact the frequency calculation is reduced to the evaluation of the fundamental frequency of a special plate associated with the real one. An extensive finite element investigation was carried out to test the accuracy of this analytical short cut method. In addition a comparative study has shown good agreement between the frequencies responses obtained from both analytical (equivalent method) and finite element solution using ANSYS program. This method allows us to both avoid huge calculations, and produce a fast and simple approximate calculus of free vibration frequencies. This in turn is a necessary part of the preliminary design phase and the general and immediate verification of a construction project to its completion.

Keywords: *Free Vibration, thin rectangular plate, isotropic, orthotropic, nodal lines position, Rayleigh's method.*

1.0 Introduction

Use of structures made of orthotropic plates requires an investigation to develop a precise and confident design. In this area, from an engineering point of view, finite element method (FEM) provides a complete solution to the problem of evaluating modes of vibration and dynamic response of an orthotropic plate when the limits of the material properties and conditions are known. However during the early stages of project design, where the main task is to select dimensions and material properties, as well as apply quality controls for the precision of the design by means of calculation by (FEM), it is very useful to have a simplified method to calculate modal frequencies of orthotropic rectangular plates. Nowadays it is possible to find a large number of contributions to the solution of this problem, where different techniques have been used and which involve the use of plates under a variety of edge and shape conditions (G.B. Warburton). Hence, it is theoretically possible to calculate exact solutions of frequency only for the case of a plate with simply supported sides. For this reason, considerable efforts are made to develop approximate methods with more accuracy. A series of articles on the dynamic behaviour of composite plates and sandwiches have been summarised by Leissa (1987) and Bert (1982).

A structure is complex if any analytical solution thereof is impossible or is obtained from delicate calculations. This definition is applied to plate's structures. And we also say that such a method is equivalent if it allows us to calculate a structure with approximate methods while not exceeding a certain percentage of error. We will therefore use these methods to find equivalent solutions for plates structures in free vibration. This method began with the article of Hearmon (1946) who initiated the study of some particular cases. Hence, to calculate the fundamental mode of an orthotropic rectangular plate with different conditions of fixity, different values of coefficients have been used by Hearmon. In addition for higher modes of modal frequencies, a special form of Rayleigh's method is proposed by Biancolini (2005). Thus evaluation of higher frequencies is reduced to the calculation of the fundamental frequency of plates under the original plate.

2.0 Problematic and Application

In this work, the problem of approximating frequencies for orthotropic plates is investigated. Based on a general formula of approximate frequency, as proposed by Hearmon, the calculation of the basic mode is shown for an orthotropic rectangular plate with different fixity conditions, using coefficient values that already exist in scientific literature (Hearmon, 1946). In addition to higher modes of frequencies, a particular form of Rayleigh's method is proposed, leading to a simple procedure for calculation of fundamental frequency. In fact, calculation of higher frequencies is reduced to the evaluation of the fundamental frequency of a specific equivalent plate associated with the actual original one.

3.0 Analysis Method:

It is important to note the following rectangular plate's characteristics:

- The nodal line is rectilinear and rectangular to the edges
- They divide the plate in sub plates that vibrate at the same value as ω.
- Nodal line presents zero displacements.

3.1. Vibratory analysis of rectangular plates with homogeneous supports:

For the first frequency of orthotropic rectangular plates, Hearmon proposed the following equation:

$$f = \frac{\lambda}{2\pi}\sqrt{\frac{D}{\rho h}} \qquad (1) \qquad\qquad \text{Where } \lambda = \sqrt{\frac{A}{a^4} + \frac{B}{b^4} + \frac{C}{a^2 b^2}} \qquad (2)$$

Where;

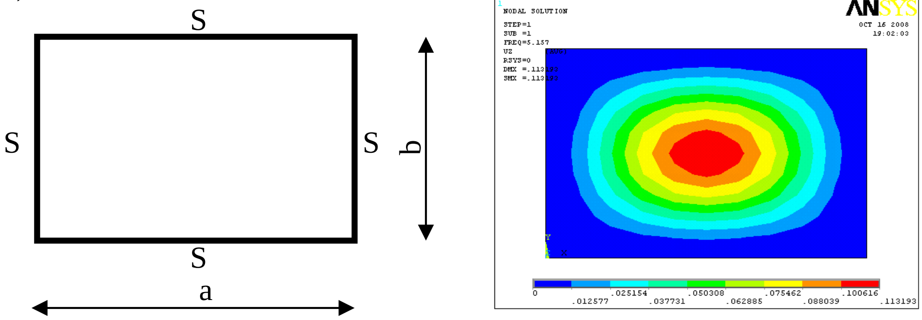
E_x, E_y: Young's modulus in bending for x and y direction respectively.
G_{xy}: Shear modulus in bending for *xy* plane.
Y_{xy} and υ_{yx} Poisson's ratio corresponding to compressive strain
x, *y*, *z* axis of the reference system

a: length of side parallel to x-axis	*b*: length of side parallel to y-axis
h: plate thickness	ρ: mass density of the material
ω: circular frequency	*f*: frequency equal to ω/2π
S: Simply supported support	*C*: Clamped support

Application of the quantitative method based on A, B, C coefficients, which are obtained from a table for the values of λ parameter (*see appendix*). In order to justify the efficiency of this method, an example of orthotropic plate of 1.5 ratios i.e. $R = \frac{a}{b} = 1.5$ for three cases of fixities SSSS, CCCC and SSCC have been calculated. Their results are showed for each case consecutively.

3.1.1. Orthotropic plate SSSS

a) Mode 1*1



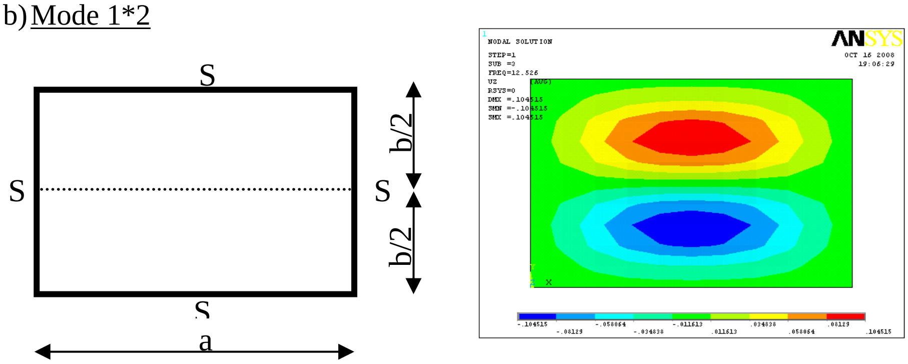
The circular frequency for mode 1*1 is obtained by the following equation:

$$\omega_{1*1} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[\pi^2\sqrt{\frac{D_x}{a^4}+\frac{D_y}{b^4}+\frac{2H}{a^2b^2}}\right] \tag{3}$$

$$D_x = \frac{E_x h^3}{12\mu} \qquad D_y = \frac{E_y h^3}{12\mu} \qquad D_{xy} = \frac{G_{xy} h^3}{12}$$

$$2H = \vartheta_{yx} D_x + \vartheta_{xy} D_y + 4\, D_{xy} \qquad \mu = 1 - \vartheta_{xy}\vartheta_{yx}$$

b) Mode 1*2

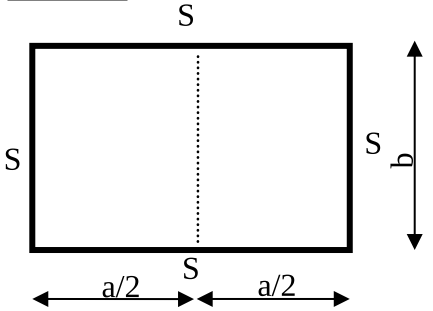


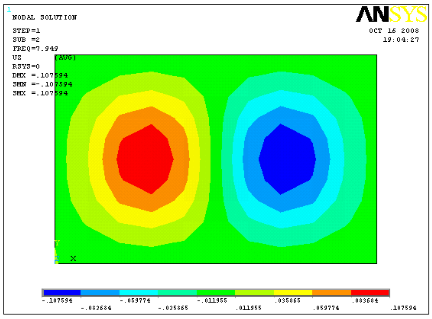
The plate develops a modal sequence with two sub plates SSSS where the nodal line position is symmetrical because of the symmetry of the extreme supports SS.

The circular frequency for mode 1*2 is obtained by the following equation:

$$\omega_{1*2} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[\pi^2 \sqrt{\frac{D_1}{(a)^4} + \frac{D_2}{\left(\frac{b}{2}\right)^4} + \frac{2H}{(a)^2\left(\frac{b}{2}\right)^2}}\right] \tag{4}$$

c) <u>Mode 2*1</u>



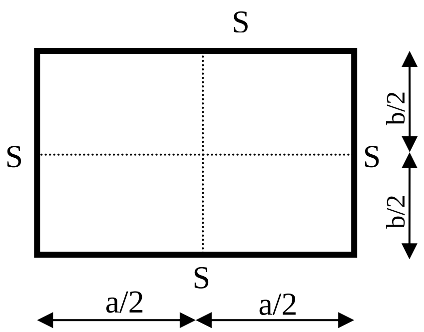


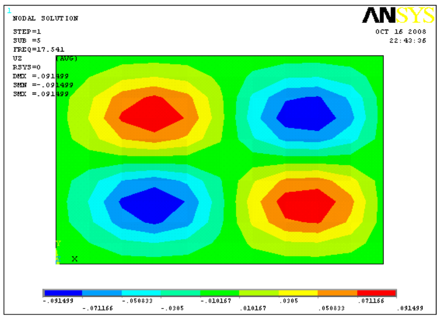
The plate develops a modal sequence with two sub plates SSSS where the nodal line position is symmetrical because of the symmetry of the extreme supports SS.

The circular frequency for mode 2*1 is obtained by the following equation:

$$\omega_{2*1} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[\pi^2 \sqrt{\frac{D_1}{\left(\frac{a}{2}\right)^4} + \frac{D_2}{(b)^4} + \frac{2H}{(b)^2\left(\frac{a}{2}\right)^2}}\right] \tag{5}$$

d) <u>Mode 2*2</u>





The plate develops a modal sequence with four sub plates SSSS where the nodal lines positions are symmetrical because of the symmetry of the extreme supports SS and SS. The circular frequency for mode 2*2 is obtained by the following equation:

$$\omega_{2*2} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(\pi)^2 \sqrt{\frac{D_1}{\left(\frac{a}{2}\right)^4} + \frac{D_2}{\left(\frac{b}{2}\right)^4} + \frac{2H}{\left(\frac{a}{2}\right)^2\left(\frac{b}{2}\right)^2}}\right] \tag{6}$$

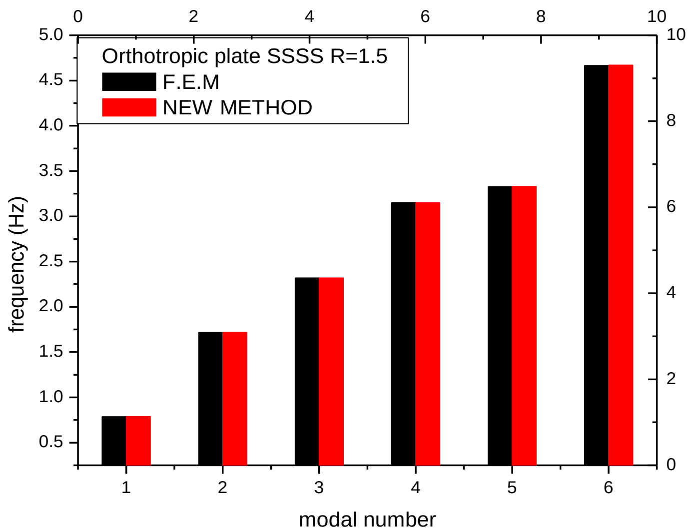
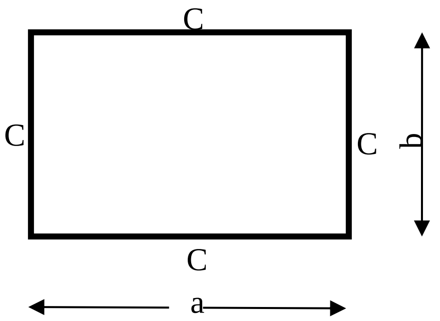
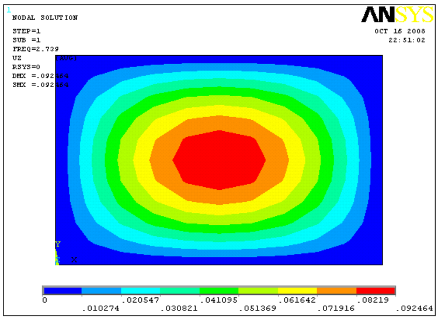


Figure 1: Comparative results of F.E.M and New Method for SSSS orthotropic plate

3.1.2. Orthotropic plate CCCC

a) <u>Mode 1*1</u>

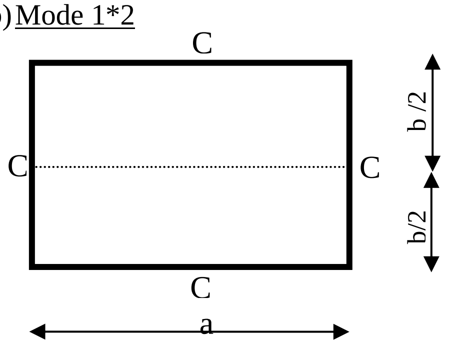


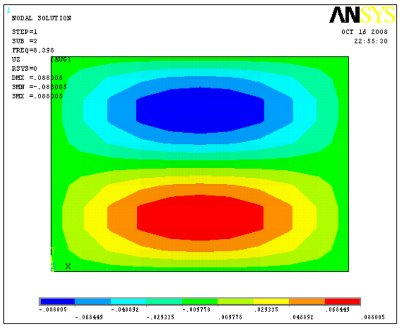


The circular frequency for mode 1*1 is obtained by the following equation:

$$\omega_{1*1} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(4.73)^2 \sqrt{\frac{D_x}{a^4} + \frac{D_y}{b^4} + \frac{0.566H}{a^2 b^2}}\right] \tag{7}$$

b) <u>Mode 1*2</u>

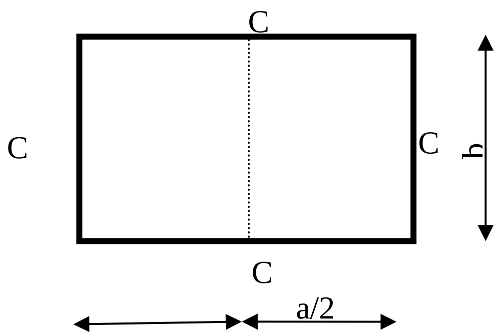


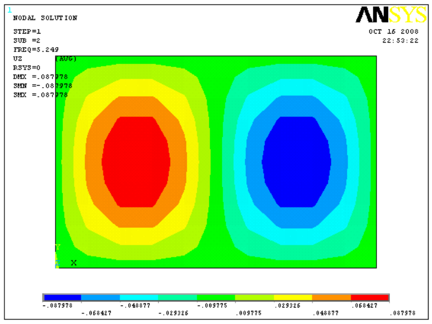


The plate develops a modal sequence with two sub plates CCSC and SCCC where the nodal line position is symmetrical because of the symmetry of the extreme supports CC. The circular frequency for mode 1*2 is obtained by the following equation:

$$\omega_{1*2} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(4.73)^2 \sqrt{\frac{D_x}{a^4} + \frac{D_y}{\left(\frac{b}{2}\right)^4} + \frac{1.115H}{a^2\left(\frac{b}{2}\right)^2}}\right] \tag{8}$$

c) Mode 2*1

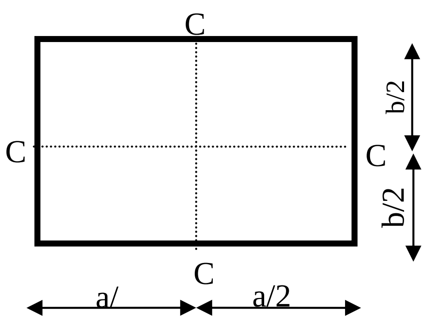


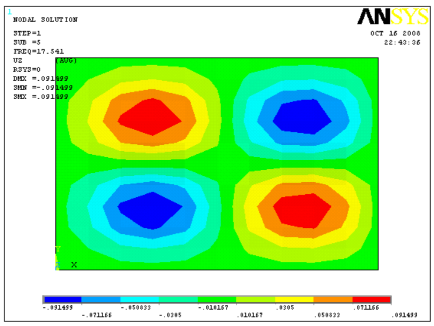


The plate develops a modal sequence with two sub plates CCCS and CSCC where the nodal line position is symmetrical because of the symmetry of the extreme supports EE. The circular frequency for mode 2*1 is obtained by the following equation:

$$\omega_{2*1} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(4.73)^2 \sqrt{\frac{0.475D_x}{\left(\frac{a}{2}\right)^4} + \frac{D_y}{b^4} + \frac{0.566H}{\left(\frac{a}{2}\right)^2 b^2}}\right] \tag{9}$$

d) Mode 2*2





The plate develops a modal sequence with four sub plates CCSS where the nodal lines positions are symmetrical because of the symmetry of the extreme supports CC and CC. The circular frequency for mode 2*2 is obtained by the following equation:

$$\omega_{2*2} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(3.927)^2 \sqrt{\frac{D_x}{a_1^{\;4}} + \frac{D_y}{b_1^{\;4}} + \frac{1.115H}{a_1^{\;2} b_1^{\;2}}}\right] \qquad (10)$$

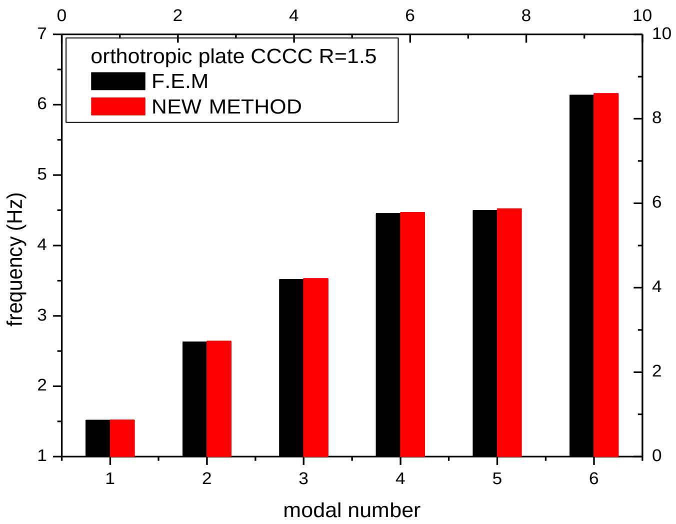
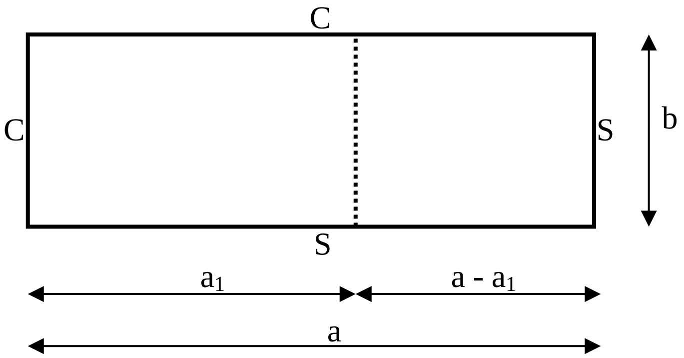


Figure 2: Comparative results of F.E.M and New Method for CCCC orthotropic plate
NOTE: This analysis can be extended to all higher modes

3.2. Vibratory analysis of rectangular plates with non-homogeneous supports:

Vibratory analysis of rectangular plates with non-homogenous support may be made in the same manner, only we are confronted by the case where extreme supports are not symmetrical and thus we will have sub plates which are not symmetrical [a_1 and ($a - a_1$) for x direction and b_1 and ($b - b_1$) for y direction]. In this case, we suppose that two sub plates vibrate at the same frequency in order to calculate the nodal line position. For example, plate SSCC for mode 2x1 will give us two sub plates SSCS and SSCC.



Sub plate SSCC

$$\omega_{2*1} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(3.927)^2 \sqrt{\frac{D_x}{a_1{}^4} + \frac{D_y}{b^4} + \frac{1.115H}{a_1{}^2 b^2}}\right] \tag{11}$$

Sub plate SSCS

$$\omega_{2*1} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(\pi)^2 \sqrt{\frac{D_x}{(a-a_1)^4} + \frac{2.441D_y}{b^4} + \frac{20333H}{(a-a_1)^2 b^2}}\right] \tag{12}$$

Because plate vibrate at the same frequency, we will have $\omega_{21[SSCC]} = \omega_{21[SSCS]}$:

$$\left[(3.927)^2 \sqrt{\frac{D_x}{a_1{}^4} + \frac{D_y}{b^4} + \frac{1.115H}{a_1{}^2 b^2}}\right] = \left[(\pi)^2 \sqrt{\frac{D_x}{(a-a_1)^4} + \frac{2.441D_y}{b^4} + \frac{20333H}{(a-a_1)^2 b^2}}\right]$$

And we will calculate a_1 value. This operation may be executed for all others non homogenous supports.

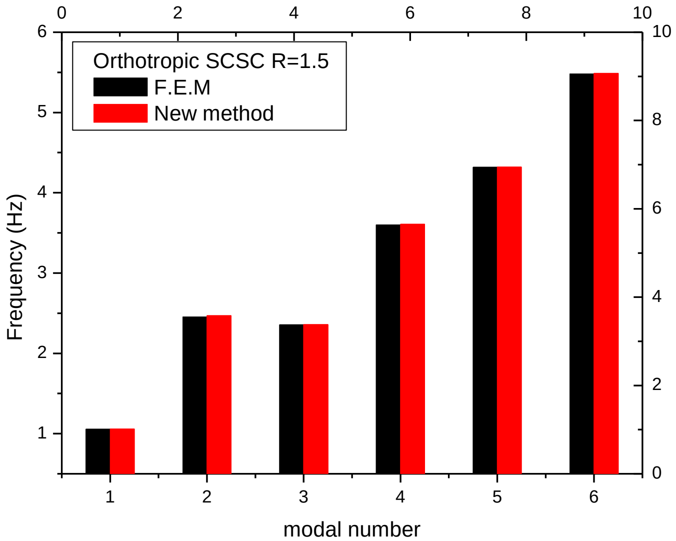


Figure 3: Comparative results of F.E.M and New Method for SSCC orthotropic plate

4.0 Conclusions and Recommendations

The equivalent method based on the principle of Hearmon method and sub-plates principle has permitted to calculate frequencies for higher modes using simple formulas. Hence calculus of frequencies for higher modes of plate is reduced to calculus of the considered sub-plates. For future recommendations, since this method is quite general for isotropic and orthotropic rectangular plates, it seems possible to extend it to force and free vibration analysis as well as others support cases.

References

Bert. C.W, “Research on dynamics of composite and sandwich plates”, Shock and Vibration Digest 14 (10) (1982).

Biancolini M. E., Butti. C, Reccia L, “Approximation solution for free vibration of thin orthotropic rectangular plates”. Journal of sound and vibrations, 2005.

Hearmon, R.F.S. "The Frequency of Flexural Vibration of Rectangular Plates with Clamped or Simply Supported Edges", Journal of applied Mechanics 26, (1959).

Leissa. A.W, “Recent research in plate vibrations”. 1981–1985—Part II: complicating effects, Shock and Vibration Digest 19 (3) (1987).

Leissa A.W. “Vibration of Plates", NASA SP-160, Scientific and technical Information Division of NASA, Washington, US, DC, 1969.

Warburton G.B. "The vibration of rectangular plates", Proceedings of the Institution of Mechanical Engineers, vol. 168, 1954.

Appendix:

Conditions of fixities	Parameter λ **(**Hearmon, 1959)
SSSS	$\lambda = \pi^2 \sqrt{\left(\frac{1}{a^4} + \frac{1}{b^4} + \frac{2}{a^2 b^2}\right)}$
CCCC	$\lambda = (4.730)^2 \sqrt{\left(\frac{1}{a^4} + \frac{1}{b^4} + \frac{0.605}{a^2 b^2}\right)}$
CSCC	$\lambda = (4.730)^2 \sqrt{\left(\frac{0.475}{a^4} + \frac{1}{b^4} + \frac{0.566}{a^2 b^2}\right)}$
SSCC	$\lambda = (4.730)^2 \sqrt{\left(\frac{1}{a^4} + \frac{0.195}{b^4} + \frac{0.485}{a^2 b^2}\right)}$
SCSC	$\lambda = (3.927)^2 \sqrt{\left(\frac{1}{a^4} + \frac{1}{b^4} + \frac{1.115}{a^2 b^2}\right)}$
SSCS	$\lambda = \pi^2 \sqrt{\left(\frac{1}{a^4} + \frac{2.441}{b^4} + \frac{2.333}{a^2 b^2}\right)}$