FOUNDATION MODELING TECHNIQUES AND CORRESPONDING JOINTED CONCRETE AIRFIELD PAVEMENT RESPONSES

Ahmed Ebrahim Abu El-Maaty* & Eman Muhammed Salah El-Din

Department of Civil Engineering, Faculty of Engineering, Menoufiaa University, Egypt

*Corresponding Author: maaty5000@yahoo.com

Abstract: Concrete pavement design procedure is expected to provide acceptable performance for a projected period under a broad range of conditions. Numerous scholars established their models grounded on finite element models (FEM) for investigation of jointed concrete pavement (JCP). FEM of concrete pavement systems requires a reasonable demonstration of the foundation layer. The main two techniques for simulating the foundation layer are the solid bricks element model with specific depth and the elastic Winkler model. Each has its advantages and limitations. The effect of these two techniques on the pavement responses was investigated using finite element code, ABAQUS (6.13). The provided model represents the real stress behavior of pavements and able to accommodate different types of rigid pavement layers, multiple combined actions, from slab-base friction, from dowel-concrete interaction, and from traffic loads. Analysis of the results is mainly in term of load transfer indicators. The notion of load transfer efficiency (LTE) is essential in airfield design procedures. Comprehending the response of rigid airfield pavement under such circumstances is essential for developing a new pavement design procedure, as well as applying appropriate preservation measures for present pavements.

Keywords: *ABAQUS, finite element, winkler, load transfer efficiency, solid element, airfield, jointed concrete pavement.*

1.0 Introduction

Rigid pavements are complex structural systems that are consist of several separate concrete slabs, longitudinal and transverse joints are provided between the slabs, which may or may not contain dowel bars. Dowel bars connect concrete slabs and transfer wheel load across the joint mainly through shear force. The significance of dowelled joints in airfields is much greater than the regular highway pavements since the applied load level on airfields is much higher and the consequences of inter–slab faulting is much greater (Jiwon & Keith, 2003). In 1926, Westergaard developed a theoretical solution for a response model of rigid pavement. Westergaard solution assumed that the slab acts as a plate applied on an elastic Winkler foundation. In a Winkler foundation, the layers underneath the concrete pavement are characterized by an infinite number of linear springs described by a single parameter, the modulus of subgrade reaction. The

All rights reserved. No part of contents of this paper may be reproduced or transmitted in any form or by any means without the written permission of Faculty of Civil Engineering, Universiti Teknologi Malaysia

stiffness of an elastic Winkler foundation is calculated as the product of the modulus of subgrade reaction, (k) multiplied by the deflection, Δ . The stiffness of an elastic Winkler foundation is assumed constant at each point under the pavement. Also the subgrade cannot transfer shear stress (Westergaard, 1926). The notion of load transfer efficiency (LTE) is simply obvious: when the load is applied to a concrete slab, stresses and deflections are decreased if a fraction of this load is conveyed to an adjoining slab. Load transfer is essential to the FAA pavement design procedure. Load transfer is a complicated phenomenon that varies with concrete material, age, environmental conditions, as thermal gradient, shrinkage and moisture content, quality of construction, magnitude and configuration of the wheel load, and the way of jointing (Ioannides, 1997).

For properly designed joints, the results suggested that a 25 % of the load transfers to the adjacent slab were an appropriate design value for load transfer (Ioannides, 1997). Analytical closed-form solutions for rigid pavement, which based on Westergaard solutions, were desirable in routine pavement analysis and design. However, the assumptions made to develop those solutions place too many limitations on the application. Due to these limitations, many computer programs have been developed to use the finite element approach to analyze, design and evaluate the performance of load transfer systems. Loading features may have an effect on the stresses and strains at the joint. In addition, the nature of the joint and pavement material properties can affect load transfer efficiency (Joannides, 1997). Variations in concrete flexural strength and elastic modulus can considerably affect the critical stresses due to aircraft loads. The field stress based LTE values for CC2 test items at NAPTF was found considerably higher than 0.25. Furthermore, the design procedure is based on static analysis assuming that the speed of the wheel is zero. However, load transfer takes place mainly under moving vehicles (Wadkar, 2009).

There are mainly two methods for a reasonable representation of the foundation layer. The main two methods for simulating the subgrade of a concrete pavement are the solid bricks element model with specific depth or the elastic Winkler model. Each has its benefits and restrictions. Although solid bricks element model with specific depth is more realistic, it is also very expensive computationally as it take much time and computing resource to converge. For the most part, modeling of the interface between the slab and foundation is more complex. On the other hand, the Winkler model is abstractly simpler and computationally effective.

However, in most pavements, the concrete slab is not bonded to the subgrade. Therefore, lift-off of the pavements is prohibited only by the self-weight of the pavements. In some cases, especially when curling or warping due to temperature gradients occurs, such lift-offs are normally encountered. (Jiwon & Keith, 2003 & Ioannides, 1997). Although these considerations, which Winkler model for foundation simulation overlook many recent studies use this technique in modeling the foundation layer in their analysis.

Maitra *et al.* (2009) examine the effects of different parameters on load transfer efficiency of at the joint with the assistance of a three-dimensional finite-element model for the analysis of a dowel-jointed concrete pavement. The subgrade was simulated as Winkler foundation in this analysis. A series of narrowly spaced, autonomous and linear spring elements were used for modeling the Winkler foundation. The normal effective stiffness of the spring element was calculated by multiplying the modulus of subgrade reaction with the influencing area of that element. The model was compared using experimental data. The group action of the dowel bar system was also inspected and useful relationships have been developed the model assumes that the subgrade fully supports the concrete slab. These may be considered as the major limitations of the model (Maitra *et al.*, 2009).

A study performed by Wadkar (2009) to determine the sensitivity of k-value, which is a representation of the pavement foundation, on load transfer efficiency of joints. The study was conducted to evaluate the averaging in k-value between two close values. The objective was to determine if a significant variation in load transfer efficiency of joint existed within the range of k-values selected for a particular value of joint stiffness. As observed in the study, no significant variation in stress based load transfer efficiency resulted from variations in the foundation stiffness values. Thus, the average k- value can be justified for further studies as input variables for 2D -FE program, JSLAB based on its effect on joint load transfer efficiency (Wadkar, 2009). Finite element models (FEM) of JCP were developed by Seo and Kim (2012) using ABAQUS considering modeling of dowel bars and the contact characteristic to concrete. The transverse stresses at joints that could cause longitudinal cracking were analyzed. Parameters affecting those cracks which contained the elastic modulus of the pavement, pavement thermal expansion coefficient, foundation stiffness, vertical thermal gradient, and bond characteristic between pavement and dowel bar were investigated. The underlying layers in JCP have been modeled employing an elastic foundation. As the foundation stiffness rises, the maximum transverse stress turns out to be higher and the stress increment ratio becomes slightly (Youngguk et al., 2013).

A study performed by Oh *et al* (2014) investigated Load Transfer Efficiency (LTE) at joints. The analysis process of the developed models contains dowel and key joints of the rigid airfield pavements to investigate behaviors under environmental and aircraft gear loads. The subgrade under the base layer has been modeled using an elastic Winkler foundation. The examination of the results showed that under the gear loads, both the dowel and key joint pavement slabs had very similar stress allocations when the gap at joint did not exist. However at the key joint case, it was found that it had larger stresses than dowelled joint case when the gap at the joint existed and the loads were applied near the joint (Oh *et al*, 2014).

This research studies the effect of foundation modeling characteristics on load transfer efficiency of indicators in JCP using the solid brick elements and elastic Winkler foundation. A 3D, nonlinear, dynamic, FEM was established using ABAQUS. This software provides numerous interactions, constraints, mesh generators, and different loading conditions, which make it suitable to carry out a complicated dynamic analysis. Creating a realistic model and calibration of model parameters is necessary for obtaining accurate dynamic and damping behavior of rigid pavements. For developing such efficient model, parameters such as element type, mesh size, interactions between foundation layers, boundary conditions, joint stiffness value and damping parameters, were accomplished in a series of steps.

2.0 Model Characteristics

2.1 Pavement Structural Model

The pavement system was selected based on a typical rigid pavement designed for usage in Egypt. The first developed model contains two dowels-jointed concrete slabs resting on base, subbase and subgrade layers as shown in Figure 1 and the second developed model consisted of two dowels-jointed concrete slabs resting on a base layer with a set of springs beneath the base layer defined by the modulus of the subgrade reaction "k" as shown in Figure 2. For avoiding difficulties related to boundary conditions, the pavement slabs were characterized by their full widths of 5.0 m with full lengths of 5.0 m. The base, subbase, and subgrade are shaped slightly wider than the slab to enable a better distribution of the stresses and widened by 0.5 m for each edge of the slab. The two adjacent slabs are connected with 14 dowel bars placed at 350 mm spacing center to center, at mid-height of the slab. The dowel bars are 32 mm in diameter and 500 mm in length, the slab thickness is 340 mm. The slabs lie on top of a 150 mm-thick of the base layer. Extension of the subbase is 250 mm. For the solid element simulation of the foundation, the extension of the subgrade is 2.5 m to ensure better simulation of subgrade responses as an approximation of the infinite foundation. The primary model has Zero gap between the two adjacent slabs to take combined effect of aggregate interlock and dowel bar as load transfer efficiencies devices (Federal Aviation Administration, 2009).

2.2 Pavement Material Models

Federal Aviation Administration uses a state-of-the-art; full-scale pavement test facility concerned with airfield pavement only at research National Airport Pavement Test Facility (NAPTF-Databases, 2016). In this study, CC6 data is used as concrete model input for the elastic behavior. MRS1 (medium-strength subgrade, rigid pavement, stabilized base) was used for modeling the pavement slabs, which allow the responses through the base layer to be more visible and easier to observe and analyze. The density is used to apply the self-weight loading on the concrete. The foundation layer in the case of solid element representation was like two separate layers with the different material

455

setting. The first layer was a subbase of Item P-209 crushed aggregate. The second layer was four cases of subgrade differ in strength: very low-strength subgrade, low-strength subgrade, medium-strength subgrade and high-strength subgrade. Data obtained from FAA report on developing FEDFAA program for rigid pavement model evaluation (Edward et al., 2007). In a Winkler foundation representation, the layers below the base layer is characterized by a series of linear springs described the modulus of subgrade reaction. The stiffness of elastic Winkler foundation for each case and the input data for solid brick simulation are detailed in Table 1 (Edward et al., 2007). The base layer was modeled by using solid brick elements. FAARFIELD program includes three items usually used in designing rigid pavement thickness, the supporting soil under the pavement slabs was modeled as represented as elastic isotropic models of Item P-301 of Soil-Cement Base was used as a Soil-Cement Base Course (Federal Aviation Administration, 2010). Dowel bars were represented using elastic isotropic material models (Federal Aviation Administration, 2010). The usage of the solid brick elements to model dowel bars correctly mimics the interaction between dowel bars and the surrounding concrete. The properties constants used are listed in Table 1.



Figure 1: Three-dimensional model of Winkle Foundation case.



Figure 2: Three-dimensional model assembly of solid brick foundation case.

Material	Parameter	Value				Parameters		
DCC alab	Modulus of	2 800 000				Modulus of	Poisson's	K
(MDS 1)	alasticity	3,000,000		Cases	Layers	elasticity	ratio	(pci)
(MKS-1)	elasticity	(psi)				(nsi)		-
	Poisson's ratio	0.15		Van lan		14 474	0.25	140
	Density	2400 kg/m^3		very low	subbase	14,474	0.35	140
L D 201	M 1 1 C	M 1 1 6 250 000 .		strength	Subgrade	4,500	0.4	
Item P-301	Modulus of	250,000 psi		Low	subbase	21,404	0.35	206
	elasticity			strength	Subarada	7 500	0.4	
	Poisson's ratio	0.2		Suchgui	Subgrade	7,500	0.4	2.10
steel bar	Modulus of	210,000 (Mpa)		Medium strength	subbase	35,429	0.35	340
	alasticity				Subgrade	15,000	0.4	
	elasticity			High	subhase	49 985	0.35	474
	Poisson's ratio	0.3		ingi	Glui	47,705	0.55	- / -
	Density	7800 kg/m^3		strength	Subgrade	25,000	0.4	

Table 1: Modelling properties.

2.3 Modeling of Interfaces

The first stage in modeling interfaces is describing contact pairs and the surfaces that could come into contact. The next stage is to specify the surfaces that interact with each other. The interface between half of the dowel bar in a slab and concrete has been modeled as a perfect bond and the other half in the neighboring slab can move along the dowel bar's axial direction. The normal behavior of the load transfer device is modeled by using hard contact pressure definition between the two surfaces. For this purpose, special surface-to-surface elements were used to model hard contact behavior. The tangential behavior of the dowel is modeled using Coulomb frictional contact between the surfaces. The different friction coefficients were taken as 0.3 for the perfectly bonded side and 0.05 for the free side of the dowel. Separation is allowed between the surfaces (Samir *et al.*, 2003).

The tangential behavior of surface between the slab and base was modeled as Isotropic Coulomb friction. According to 1993 AASHTO pavement design guide, a range of 0.9 and 2.2 of the friction coefficient between the slab and base interface and its variance rely on the base type. For this study, it was assumed 1.5. Loss of contact between slab and base is modeled using normal hard contact that allows the surfaces to separate after coming in contact (Huang, 2003). No separation is allowed between foundation layers in the case of the solid bricks simulation method. The interaction between the top of the subbase and the upper surface of subgrade were simulated by the use of hard contact interface elements and isotropic Coulomb friction model, the coefficient of friction was assumed 1.5 (Huang, 2003).

2.4 Meshing of the Model

Meshing irregularities in the model in a non-uniform way can create stresses does not exist in real structures. Wedge elements with very fine meshing were selected for the dowel bars to ensure the regular distribution of mesh element around dowel bar. First order 6-node linear triangular prism elements are used for the dowel bars meshing (ABAQUS User's Guide, 2013). Reduced-integration elements lean to be in some way more efficient. At lower computational cost, outputs are frequent as effective as or better than full integration. So, eight-node linear continuum three-dimensional brick element (C3D8R), reduced integration, and hourglass control available in ABAQUS (6.13) are used for discretizing the concrete slabs. Recognizing that the joints, the area around the dowels and loading path are critical stress zones that can initiate pavement failure, a refined mesh was developed in these regions, to capture the flow of stresses accurately around the dowel bars. These elements have the capability of simulation of large deformation, geometric and material nonlinearity. All layers of the payement (base, subbase and subgrade) are simulated with the same element type to preserve the continuity of nodes between successive layers. Figure 3 demonstrations the cross section at the pavement joint and its meshing details.



Figure 3: Mesh details through the dowelled pavement joint.

2.5 Aircraft Loading

The moving tire is to be modeled as tire imprint area to represent a smooth pavement surface. The slabs are loaded using a single wheel aircraft at the joint and the load transfer efficiencies are calculated using the stresses and strains at the loaded and unloaded slabs. A single wheel aircraft, F-15, with an edge loading case is used for this analysis. The main characteristics of the aircraft are shown in Table 2 and Figure 4 (Federal Aviation Administration, 2010). Dynamic applications are mainly divided into

three classes: transient fidelity, moderate dissipation applications and quasi-static applications. In this study, transient fidelity approach, which founded on the notion of moving the tire print patch at successive locations along the pavement for each step time, are used. Damping effect is considered in both analysis types.

ABAQUS offers "Rayleigh" model to simulate the damping effect. It offers a suitable concept to damp minor (mass dependent) and upper (stiffness-dependent) frequency range behavior. The pavement damping is mainly stiffness proportional and hence the Rayleigh coefficient β is considered to define damping with a value of 0.2. Previous studies show that dynamic LTE(s) is not sensitive foundation damping and hence not used in this model (Xinhua et al., 2010).

For avoiding large over closures in contact problems, adding additional steps to the analysis is required, which minimize convergence difficulties and make the solution far more efficient. ABAOUS/CAE creates a special initial step at the beginning of the model's step sequence. The initial step allows defining boundary conditions, predefined fields, and interactions at the very beginning of the analysis. The initial step is followed by one or more analysis steps. The second step in all cases in this study is set for applying gravity loads and stabilization of the model; the step type is general/static. The model then evolves in a sequence of steps as it responds to the loads defined in each step. Dynamic loading can be modeled by using shifted tire loading imprint area over the pavement surface across the joint. Loading moving step by step was used to simulate wheel motion in a determined speed. The dynamic implicit analysis method is used to calculate the transient response of the aircraft (ABAQUS User's Guide, 2013).

	Tire pressure	Footprint	Tire contact	Tire contact	Dual spacing	Tandem
Aircraft	'Mpa'	area mm²	length 'mm,'	width 'mm,'	'mm,'	spacing 'mm,'
F-15	2.344	61290	353.4	220.8	0.0	0.0

Table 2: Main characteristics of the aircraft.





Figure 4: Single load of F-15

3.0 Models Results

The values of any variable along steps and frames were displayed using contour plots and charts to show the value of such variable at a particular step of a model in the certain model database. Output requests in this study mainly focus on showing stresses around deformed dowel hole at the location of maximum stresses at end and beginning of load application for each step of the entire analysis history. They also focus on showing stresses and deformation at the critical edge of the loaded and unloaded slab. The history of a certain variable would be plotted versus the time of the moving axle (the change in loading position) from a certain point to the joint. Figures 5 and 6 show general pavement results obtained using the developed model. The most efficient process of examining the precision of the developed model is to match its results with field test measurements for the same arrangement under the same loading circumstances. Due to the limited resources and absence of the ability to perform a specialized test procedure, the verification process is done by using the results from using the HWD test on NAPTF sections by comparison of its results to the developed model which was documented in a previous study (Abu El-Maaty *et al.*, 2016).



Figure 5: Dowel deformation across the joint.



Figure 6: Stresses distribution at the surrounding of the dowel bar across the joint.

459

3.1 Effect of Foundation Material Properties

The representation of the foundation layers was two separate layers with the different material setting. The first layer was a subbase of Item P-209 Crushed Aggregate. The second layer was four cases of subgrade differ in strength: Very Low-Strength Subgrade, Low-Strength Subgrade, Medium-Strength Subgrade and High-Strength Subgrade. The supporting soil was modeled as an elastic material. Item P-301 was used as a Soil-Cement Base Course and MRS1 was used for modeling the pavement slabs, which allow the responses of the base layer to be more visible and easier to observe and analyze. A sensitivity study is performed to understand the effect of foundation layer material on load transfer indicators. The observations of the stresses and strains histories at the transverse joint and the middle of the slab showed significant changes in stresses and strains and their distribution.

The deflection based load efficiency is decreased significantly with the increase of foundation stiffness. The change from very low case to high case causes a change about 15.3 % in deflection based load efficiency. On the other hand, it was observed that the stress based load transfer efficiency is not sensitive foundation layers stiffness. The LTE (S) almost remains the same from case to case. Although the insignificance of the foundation layers' stiffness effects on the stresses, it was observed that the stresses are slightly decreased. However, the stress based load transfer is increased with the increase of the foundation stiffness. This can be attributed to the little amount of the stresses, which is transferred through the foundation layers, not across the joint. Figures 7 to 9, summarizes the behavior indicator observed in these models.



Figure 7: Deflection-based load transfer efficiency across the joint.



Figure 8: Stress based load transfer efficiency across the joint.



Figure 9: Relation between load transfer indicators across the joint.

3.2 Comparison between the Foundation Modeling Techniques

FAA Advisory Circular (2009) provides instruction curves to determine the modulus of subgrade reaction (k). The foundation stiffness is first recognized based on subgrade CBR value that is dependent on the modulus of elasticity when the thicknesses are fixed. Therefore, for a fixed thickness, the stiffness of the foundation is dependent on its modulus of elasticity. A sensitivity study is performed to understand the effect of foundation layer material on load transfer indicators. In this case, the two layers that represent the foundation are replaced by a set of springs beneath the base layer defined by the modulus of the subgrade reaction "k". Results show that increasing the

foundation modulus is significantly reduces the strains, which follows the relation: $p = k \Delta$ Where (k) modulus of subgrade reaction (psi), p is reactive pressure (psi) and $\Delta =$ deflection of the slab (in.). The supporting capability of the foundation layers is its stiffness, which is in consistence with the previous modeling technique that depended on defining the foundation layers on their modulus of elasticity.

Examination of the results shows that in the two cases of foundation modeling method, the deflection based load efficiency decreased significantly with the increase of foundation stiffness and the stress based load transfer efficiency is not sensitive foundation layers stiffness. As the verification process documented in (Abu El-Maaty et al., 2016) show, the model which uses solid bricks element to model the foundation layer can be the base of comparison and evaluation for the Winkler foundation modeling method. The average difference in the values of stress based load transfer between solid brick model and Winkler model was about 99.33%. On the other hand, the average difference in the values of deflection based load transfer between solid brick model and Winkler model was about 94 % as shown in the Figures 10 and 11. The examination of the values of the stresses which used to calculate the stress based LTE shows that the average difference in the values of stresses between solid brick model and Winkler model was about 97.05% at the loaded slab and the unloaded slab was about 95.7% as shown in the Figures 12 and 13. On the other hand, the average difference in the values of deflections used to calculate the deflection-based LTE between solid brick model and Winkler simulation case was about 82% in the loaded slab and in the unloaded slab was about 78% as shown in the Figures 14 and 15. The Winkler foundation is good simulation technique as an indicator on the values of load transfer efficiency and stresses but less reliable in simulating the exact response of the rigid pavement in term of deflections.

The numerical solution provided by model tends toward a unique value as the mesh density is increased. The computer resources required to run simulation also increased, as the mesh is refined. The mesh is called converged when further mesh refinement produces negligible changes in the solution accuracy. As the mesh density increases, the CPU time and the storage required running the simulation for the analysis increases. Removing the subgrade and subbase layer and replacing them with Winkler foundation reduce the total element number of the model by 27.8%. However, the average reduction in convergence time was about 7%. It was found that Winkler foundation was easier to converge during the static step and was as difficult as the solid brick case in term of convergence. Getting converged solutions for highly nonlinear models can from time to time be challenging. Complications can arise, especially in simulations involving contact, dynamic behavior, complicated material models and geometrically unstable behavior. Further increases in mesh density require more advanced computing resources without a significant advantage in result accuracy. The model should not be more composite than needed to define the behavior of concern. It appears rational to reduce the complexity of a simulation by supposing it acts statically when the procedure is slow.

463

However, this simplification can make the model harder to solve in the case of analysis of dynamic applications.



Figure 10: Deflection based load transfer efficiency across the joint.



Figure 11: Stress based load transfer efficiency across the joint.



Figure 12: Stresses at the loaded slab across the joint.



Figure 13: Stresses at the unloaded slab across the joint.



Figure 14: Deflection at loaded slab across the joint.



Figure 15: Deflection at unloaded slab across the joint.

4.0 Conclusions

Results show that increasing the foundation modulus is significantly reduces the strain; the deflection based load efficiency is decreased significantly with the increase of foundation stiffness. On the other hand, it is observed that the stress based load transfer efficiency is not sensitive foundation layers stiffness. The LTE (S) almost remains the same from case to case. The usage of the solid brick elements to model foundation outperforms Winkler foundation as it correctly mimics the interaction between the base layer or the concrete slab and the foundation of the pavement. The Winkler foundation is good simulation technique as an indicator on the values of load transfer efficiency and stresses but less reliable in simulating the exact response of the rigid pavement in term of deflections. Different subgrade modeling techniques were investigated in term of accuracy and convergence time. These techniques are solid bricks element with a certain depth, the usage of Winkler foundation modeling. Winkler foundation model cost less element and less time to converge especially in the static application of the load. However, in the dynamic application, Winkler foundation model was found less efficient in term of accuracy and required convergence time than solid bricks foundation. The usage of the solid brick elements to model dowel bars correctly mimics the stress and strains patterns in the subgrade. Modeling the subgrade as a Winkler foundation does not permit the investigation of distress modes in the subgrade. The foundation layers are commonly known as their nonlinear behavior, any support added to the foundation layer such as different applications of Geosynthetics can be examined easily using solid bricks elements modeling.

References

ABAQUS User's Guide, (2013). ABAQUS User's Guide : Dassault Systèmes. Simulia Corp.

- Abu El-Maaty, A.E., Ghada, M.H., and Eman M. S. (2016). Modeling of dowel jointed rigid airfield pavement under thermal gradients and dynamic loads. Civil Engineering Journal, Vol. 2, No. 2, pp 38-51.
- Advisory Circular (2009). AC Report No: 150/5320-6E Federal Aviation Administration U.S. Department of Transportation, Airport Pavement Design and Evaluation.
- Edward, G., and Izydor K. (2007). FAA finite element design procedure for rigid pavements. Report for U.S. Department of Transportation Federal Aviation Administration, 2007.
- Federal Aviation Administration, (2010). FAARFIELD 1.305 program database. [http://www.faa.gov/airports/engineering/design_software/].
- Huang Yang, (2003). Pavement analysis and design. Second edition ,Prentice Hall, 2003.
- Wadkar A., (2009) . Study of load transfer efficiency of airfield rigid pavement joints based on stresses and deflections. M.S., Civil and Environmental Engineering, Henry Rowan College.
- Ioannides, M. (1997). Advanced pavement design: finite element modeling for rigid pavement joints. Report I, U.S. Department of Transportation Federal Aviation Administration.
- Jiwon, K., and Keith, H. (2003). Three-dimensional finite element analysis of doweled joints for airport pavements. Transportation Research Record: Journal of the Transportation Research Board, Volume 1853.
- Maitra, S. R., Reddy, K. S., and Ramachandra, L. S. (2009). Load transfer characteristics of dowel bar system in jointed concrete pavement. Journal of Transportation Engineering. 135(11), 813–821.
- NAPTF-Databases (2016). National airport pavement test facility. U.S. Department of Transportation, Federal Aviation Administration.
- Oh, H. J., Kim, S., Chun, W., Lee, Y. H., and Young K. (2014). Effect of joint type on rigid airfield pavement behavior. KSCE Journal of Civil Engineering, Volume 18, Issue 5, pp 1389-1396.
- Samir, S., William, G., Mourad, Y., Motamarri, S. (2003). Effect of bonding force on stresses in concrete slabs. West Virginia University, Department of Civil and Environmental Engineering, Morgantown, TRB.
- Westergaard, H.M. (1926). Analysis of stresses in concrete pavements due to variations of temperature. Highway Research Board, Vols. Vol. 6: 201-215.
- Xinhua, Y.U. and Xiaochun,W. (2010). Considering joint load transfer efficiency of rigid pavement dynamic effects under a single moving load . 2nd International Conference of Information Engineering and Computer Science (ICIECS), pp.1-4, IEEE publisher, Wuhan.
- Youngguk, S., and Seong, M.K. (2013). Longitudinal cracking at transverse joints caused by dowel bars in jointed concrete pavements. KSCE Journal of Civil Engineering, Volume 17, Issue 2, pp 395-402.