

TECHNICAL NOTE

TESTING THE ACCURACY OF SEDIMENT TRANSPORT EQUATIONS USING FIELD DATA

Hydar L. Ali*, Thamer Ahamed Mohammed, Badronnisa Yusuf & Azlan A. Aziz

Department of Civil Engineering, Faculty of Engineering, Universiti Putra Malaysia, 43400 Serdang, Selangor, Malaysia

*Corresponding Author: *hydar_e@yahoo.com*

Abstract: In order to recommend the equations that can accurately predict sediment transport rate in channels, selected sediment transport equations (for estimating bed load and suspended load) are assessed using field data for 10 rivers around the world. The tested bed load equations are Einstein, Bagnold, Du Boys, Shield, Meyer-Peter, Kalinskie, Meyer-Peter Muller, Schoklitsch, Van Rijn, and Cheng. Assessment show that Einstein and Meyer-Peter Muller equations have the least error in their prediction compared with the other tested equations. Based on the field data, each of Einstein and Meyer-Peter Muller equations gave the most accurate bed load estimations for three rivers while Schoklitsch equation and Du boys equation gave the most accurate bed load estimations for two rivers and one river respectively. The lowest values of Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) were obtained from the applying Einstein equation for estimating bed load for Oak Creek River and these values were found to be 0.02 and 0.04 respectively. On the other hand, the tested equations for predicting suspended load are Einstein, Bagnold, Lane and Kalinske, Brook, Chang, Simons and Richardson, and Van Rijn. Among the above tested equations, assessment show that Bagnold, Einstein and Van Rijn gave the most accurate estimation for the suspended load. The lowest values of Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) were obtained from applying Bagnold equation and these values were found to be 0.012 and 0.015 respectively.

Keywords: *Sediment transport equations, river, application, assessment, testing*

1.0 Introduction

Sediment is defined as the grainy material transported as particles with range of sizes that originally came from physical or chemical degradation of rocks by flow from the basin (Van Rijn, 1993; Yang, 2010). Sedimentation involves the processes of erosion, entrainment, transportation, deposition and compaction (Graf, 1971). Sediment causes many problems such as reducing storage capacity of rivers and reservoirs, effect water quality, problems in operating turbines and pumping stations, and erosion and

sedimentation at hydraulic structures. Therefore, it is important to study sediment transport in channels. On the other hand, calculating sediment loads is not easy to obtain (Ab. Ghani *et al.*, 2010). The sediment load can normally be examined on the basis of sediment source, methods of sediment transport, or measurement method. The sediment sources are identified as a bed material load and wash load (fine particles not found in the bed). The methods of sediment transport are classified as either in suspension or near the bed. The mechanism of sediment transport has been a subject of study for decades due to its importance. To date, there are many available equations for calculating sediment discharge in alluvial channels and basically these equations are of three types, i.e., bed load, suspended load, and total load equations. The later can be obtain directly by empirical relations or indirectly by summation of the bed load and suspended load, which are omputed separately using appropriate bed-load and suspended-load equations. This method contradicts the observation of natural flowing conditions, where no sharp distinction between the bed and suspended loads. The categories of bed load and suspended load are not rigid and this is arributed to the mangnitude of velocity and the resulting turbulence in the open channel. For instance, in high velocity or very turbulent water, gravels and large size of sediment can travel most of them in suspension. On the other hand, in very low velocity or very low turbulent, the small size of sediment particles such as silt and clay move totally in bed load (Chien and Wan, 1999). However, the main objective of this paper is to validation of several sediment transport equations using field data of 10 rivers around the worldand to recommend equations with the most accurate predictions.

2.0 Methodology

A total of 16 different equations for estimating sediment transport (bed load and suspended load) were tested using reliable data for 10 rivers located at different parts of the world. The tested bed load equations are Einstein, Bagnold, Du Boys, Shield, Meyer-Peter, Kalinskie, Meyer-Peter Muller, Schoklitsch, Van Rijin, and Cheng while the tested suspended load equations are Einstein, Bagnold, Lane and Kalinske, Brook, Chang, Simons and Richardson. Results from these equations are statistically tested to recommend the most accurate equations. The methodology is summarized in the flow chart shown in Figure 1.

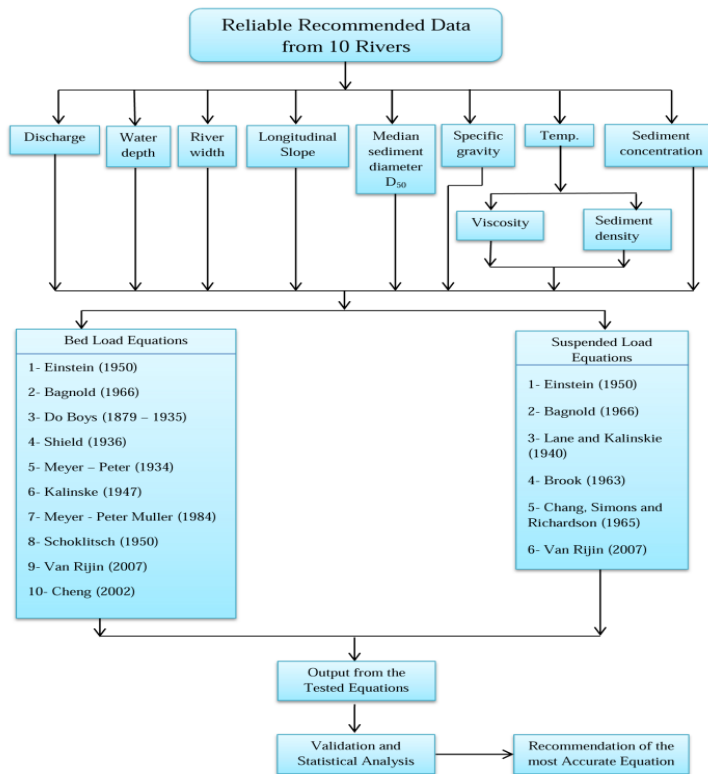


Figure 1: Flowchart represents the methodology

2.1 Sediment Field Data

Sediment field data of 10 rivers around the world are selected from published literatures (Brownlie, 1981). Due to the recommendations made on their reliability, the field data have been used for testing the accuracy of the selected sediment transport equations. Table 1 shows the summary of these data. The following are names of the rivers from which the field data are collected.

- 1- Indian canal Data of Chaudry *et al.* (1970).
- 2- Colorado River Data of the U.S. Bureau of Reclamation (1958).
- 3- Middle Loup River Data of Hubbell and Matejka (1959).
- 4- Mississippi River Data of Toffaleti (1968).
- 5- Niobrara River Data of the Colby and Hembree (1955).
- 6- Oak Creek Data of Milhous (1973).
- 7- Portugal River Data of Da Cunha (1969).
- 8- Rio Grande Conveyance Channel Data of Culbertson *et al.* (1976).
- 9- Snake and Clearwater River Data of Seitz (1976).
- 10- Trinity River Data of Knott (1974).

Table 1: Summary of field data

No	River name	Range of median particle diameter (mm)	Range of discharge (m ³ /s)	Range of river width (m)	Range of velocity (m/s)
1	Indian canal Data	0.09 – 0.19	109.6 – 424	55.474 -118.262	0.363 - 1.259
2	Colorado	0.236 – 0.36	83.34 – 500.16	92 – 254.55	0.363 – 1.259
3	Middle Loup	0.275-0.395	9.373-11.723	42.977-46.33	0.638-0.94
4	Mississippi	0.165-0.342	22851-28826	1097.3-1109.5	1.344-1.609
5	Niobrara	0.218-0.351	6.456-16.055	21.164-21.946	0.688-1.271
6	Oak Creek	8.2-26	1.416-3.397	5.37-5.914	0.807-1.118
7	Portugal	2.603	59.598-194.094	102-183	0.785-0.973
8	Rio Grande	0.18-0.28	15.857-39.077	20.422-22.86	0.805-1.518
9	Snake and Clearwater	0.52-33	1832-3511.2	176.784-198.12	2.377-2.997
10	Trinity	3.4-11.8	39.642-82.683	30.175-53.95	1.265-2.177

2.2 Bed Load Equations

There are many available equations for estimating bed load in channels and these equations where based on different concepts. In this study, only 10 of these equations were applied to estimate the bed load using field data. Table 2 shows these equations.

Table 2: The selected bed load equations

Equation name	Concept	Equation
Einstein, 1950	Probabilistic defined as the rate of erosions equals the rate of depositions	$q_{b,w} = \phi * \gamma s \sqrt{\frac{D_{50}^3 * g(\rho s - \rho)}{\rho}}$ Eq.(1)
Bagnold, 1966	power concept, its production of the available stream power and efficiency	$q_{b,w} = \frac{P}{B} \frac{e_b}{\tan \alpha} \left[\frac{\gamma}{\gamma_s - \gamma} \right]$ Eq.(2)
Du Boys, 1879 and Straub, 1935	shear stress approach	$q_b = k_3 \tau (\tau - \tau_c)$ Eq.(3)
Shield, 1936	shear stress approach	$q_{bv} = \frac{10 q s_0 \rho^2 (\tau - \tau_c)}{\rho_s (\rho_s - \rho)^2 g D_{50}}$ Eq.(4)
Meyer – Peter, 1934	energy slope approach	$\frac{0.4 q_{bw}^{2/3}}{D_{50}} = \frac{q_w^{2/3} S}{D_{50}} - 17$ Eq.(5)
Kalinske, 1947	shear stress approach	$\frac{q_{bv}}{U_* D} = f \left[\frac{\tau_c}{\tau_0} \right]$ Eq.(6)
Meyer – Peter Muller, 1948	energy slope approach	$\gamma \left[\frac{k}{kr} \right]^{3/2} RS = 0.047 (\gamma_s - \gamma) D_{50} + 0.25 \left[\frac{\gamma}{g} \right]^{1/3} \left[\frac{\gamma_s - \gamma}{\gamma} \right]^{2/3} q_{bw}^{2/3}$ Eq.(7)
Schoklitsch, 1950	discharge approach	$q_{bw} = 2500 s^{3/2} (q - q_c)$ Eq.(8)
Van Rijn, 2007, a	analytical relationship	$q_{bw} = 0.015 \rho_s V d \left(\frac{D_{50}}{h} \right)^{1.2} M_e^{1.5}$ Eq.(9)
Cheng, 2002	An exponential equation is not including the concept of critical shear stress	$q_{bw} = \left[\Phi * D_{50} \sqrt{D_{50} * g * (s - 1)} \right] * \rho_s$ Eq.(10)

2.3 Suspended Load Equations

There are many available equations for estimating suspended load in channels and only six of them were applied to estimate the suspended load using field data. Table 3 shows these equations.

Table 3: The selected suspended load equations

Equation name	Concept	Equation
Einstein, 1950	The concepts of these equations are based on the exchange theory under equilibrium conditions and velocity distribution with some other consumptions	$q_{s,w} = 11.6 u_* c_a a [Pe I_1 + I_2]$ Eq.(11)
Bagnold, 1966		$q_{s,w} = e_s (1 - e_b) \frac{\rho_s V}{B \omega} \left[\frac{\gamma}{\gamma_s - \gamma} \right]$ Eq.(12)
Lane and Kalinske, 1941		$q_{s,w} = q C_a P_L e^{15 \omega a / d U^*}$ Eq.(13)
Brook, 1963		$\frac{q_{s,w}}{q C_{md}} = T_B \left(\frac{k V}{U^*}, Z_1 \right)$ Eq.(14)
Chang, Simons and Richardson, 1965		$q_{s,w} = d C_a \left(V I_1 - \frac{2 U^*}{k} I_2 \right)$ Eq.(15)
Van Rijn, 2007, b		analytical relationship $q_{s,w} = 0.008 \rho_s V D_{50} M_e^{2.4} (D^*)^{-0.6}$ Eq.(16)

2.4 Validation and Statistical Method

Two methods are used to compare the performance of the tested equations and these methods are Mean Absolute Error (MAE), and Root Mean Square Error (RMSE). The methods are described by the following equations:

$$MAE = \frac{1}{N} \sum_{N=1}^N |Q_{sm} - Q_{sc}| \quad (17)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{N=1}^N (Q_{sm} - Q_{sc})^2} \quad (18)$$

where, N is the number of data sets, Q_{sm} is the measured suspended load and Q_{sc} is the calculated suspended load.

3.0 Results and Discussion

3.1 Comparisons of Bed Load Equations

Hydraulic, sediment and morphological data for 10 rivers around the world were used to estimate the bed loads at different sections (92 sections). The Equations used in estimating the bedload are (1), (2), (3), (4), (5), (6), (7), (8), (9), and (10). Samples of

results which are obtained from applying Equations (1) to (10) are shown in Tables 4 and 5.

Table 4: Predicted bed load discharges and measured in (kg/s/m) for Indian canal data

Kalinske (1947) (Eq.(6))	Meyer peter Muller(1948) (Eq.(7))	Schoklitsch (1950) (Eq.(8))	Cheng (2002) (Eq.(10))	Du boys (1879) (Eq.(3))	Shield (1936) (Eq.(4))	Meyer Peter (1934) (Eq.(5))	Einstein (1950) (Eq.(1))	Bagnold (1966) (Eq.(2))	Van Rijin (2007) (Eq.(9))	q _{bw} measured
0.0235	0.5312	0.0103	0.2496	0.4165	2.8197	0.0149	0.0602	0.0306	0.0621	0.3549
0.0386	0.3049	0.0154	0.6127	1.6443	6.9272	0.0225	0.0235	0.0434	0.0229	0.3944
0.0217	0.1307	0.0034	0.1717	0.2155	0.9167	0.0042	0.0038	0.0147	0.0095	0.2763
0.0452	0.2512	0.0154	0.7367	2.0478	6.9024	0.0224	0.0136	0.0458	0.0159	0.2924
0.0524	0.2714	0.0191	0.8907	2.5481	8.5830	0.0277	0.0216	0.0524	0.0167	1.0230
0.0385	0.5012	0.0117	0.4023	0.7969	4.3104	0.0165	0.1171	0.0427	0.0597	1.6920
0.0259	0.8912	0.0101	0.2274	0.3289	2.5463	0.0141	0.2469	0.0372	0.1442	1.7126
0.0293	0.8438	0.0108	0.2569	0.4029	2.6397	0.0151	0.2090	0.0388	0.1299	2.2083
0.0436	1.1335	0.0183	0.4226	0.7705	5.1997	0.0262	0.4281	0.0583	0.1680	4.3707
0.0208	0.2598	0.0022	0.0708	0.0396	0.2558	0.0019	0.0675	0.0119	0.0386	0.2841

Table 5: Predicted bed load discharges and measured in (kg/s/m) for Colorado River data.

Kalinske (1947) (Eq.(6))	Meyer peter Muller(1948) (Eq.(7))	Schoklitsch (1950) (Eq.(8))	Cheng (2002) (Eq.(10))	Du boys (1879) (Eq.(3))	Shield (1936) (Eq.(4))	Meyer Peter (1934) (Eq.(5))	Einstein (1950) (Eq.(1))	Bagnold (1966) (Eq.(2))	Van Rijin (2007) (Eq.(9))	q _{bw} measured
0.06392	0.20432	0.00652	0.33082	0.32001	0.780332	0.00642	0.023078	0.022739	0.0148778	0.05157
0.094042	0.41004	0.011749	0.45445	0.44598	1.319644	0.01297	0.320904	0.036178	0.0421138	1.507685
0.048684	0.21936	0.005474	0.21564	0.18774	0.505366	0.00547	0.002087	0.018022	0.0174406	0.404973
0.105978	0.80631	0.036392	1.20987	2.54099	9.170279	0.05175	0.611373	0.087401	0.0797487	0.489615
0.125688	1.00372	0.049336	1.58527	3.69958	13.77095	0.07136	0.917789	0.113567	0.1012479	1.253026
0.063416	0.27717	0.010147	0.42769	0.58504	1.558988	0.01249	0.020466	0.02974	0.0211607	0.168085
0.029663	0.33578	0.002701	0.06876	0.02569	0.176902	0.00158	0.128406	0.01313	0.0542097	1.217224
0.095084	0.63065	0.019712	0.70677	0.94233	3.425278	0.02546	0.488876	0.058003	0.0707102	0.403968
0.107892	0.35413	0.022131	1.03751	1.79533	4.617155	0.02958	0.104217	0.056142	0.0242489	0.643064

Two statistical methods [Equation (17) and Equation (18)] are used to test the performance of bed load equations by comparing the predicted and measured values. Table 6 shows results of the statistical test while Figures 2 to 11 also show the comparisons between computed and observed bed loads for each river.

Table 6: Summary of the results obtained from of the statistical tests for bed load equations

No:	Name of river	Equation name	MAE	RMSE
1	Oak Creek	Einstein	0.02	0.04
2	Middle Loup River	Einstein	0.19	0.24
3	Niobrara River	Einstein	0.22	0.29
4	Indian canal	Meyer-Peter Muller	0.784	1.227
5	Rio Grande	Meyer-Peter Muller	1.04	1.35
6	Colorado River	Meyer-Peter Muller	0.35	0.48
7	Portugal River	Schoklitach	0.02	0.03
8	Snake and Clearwater River	Schoklitach	1.04	1.011
9	Trinity River	Meyer-Peter	0.26	0.32
10	Mississippi River	Du boys	2.15	2.38

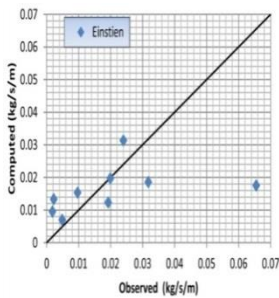


Figure 2. Graphical comparison between observed and computed bed load for Oak Creek River using Einstein (1950)

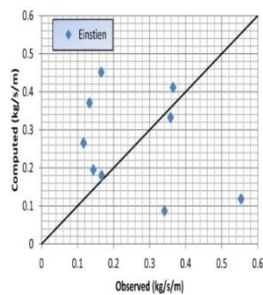


Figure 3. Graphical comparison between observed and computed bed load for Middle Loup River using Einstein (1950)

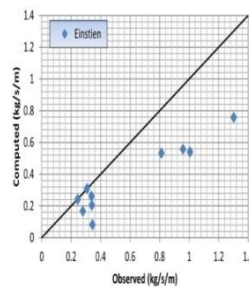


Figure 4. Graphical comparison between observed and computed bed load for Niobrara River using Einstein (1950)

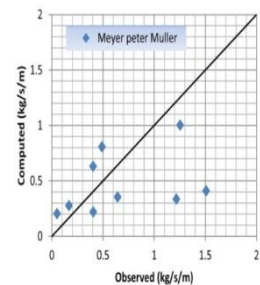


Figure 5. Graphical comparison between observed and computed bed load for Colorado River using Meyer Peter-Muller (1948)

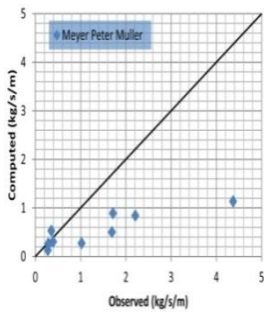


Figure 6. Graphical comparison between observed and computed bed load for Indian canal data using Meyer peter-Muller (1948)

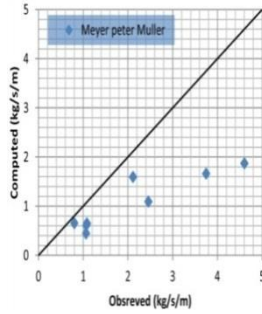


Figure7. Graphical comparison between observed and computed bed load for Rio Grande River using Meyer peter-Muller (1948)

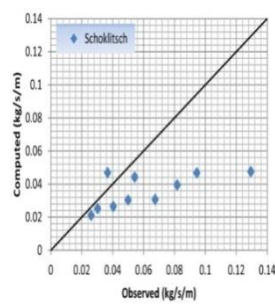


Figure 8. Graphical comparison between observed and computed bed load for Portugal River using Schoklitsch (1950)

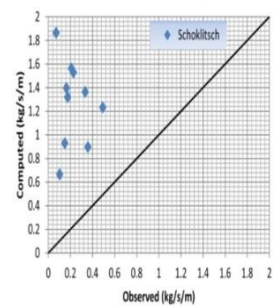


Figure 9. Graphical comparison between observed and computed bed load for Snake Clearwater River using Schoklitsch (1950)

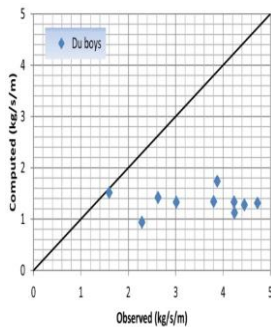


Figure 10. Graphical comparison between observed and computed bed load for Mississippi River using Du boys (1879)

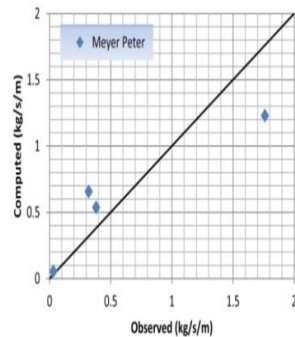


Figure 11: Graphical comparison between observed and computed bed load for Trinity River using Meyer Peter (1934)

Figures 2 to 11 and Table 6 show that Einstein and Meyer-Peter Muller equations have least error compared with the other tested equations. These equations gave the most accurate bed load estimation for three rivers while Schoklitsch equation and Du boys equation gave the most accurate bed load estimations for two rivers and one river respectively. The least values of Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) were found to be 0.02 and 0.04 respectively. This was associated with applying Einstein equation for Oak Creek River. The Graphical comparison show that computed values for the bed load for rivers Oak Creek, Middle Loup and Colorado are scattered around the line of perfect agreement while the majority of the applied equations gave under prediction except the computed bed loads for Snake and Clearwater River gave completely over prediction compared with the field data.

3.2 Comparisons of Suspended Load Equations

The selected equations assessed for predicting suspended load are Equations (11), (12), (13), (14), (15), and (16) and sample of the results obtained from applying these equations are shown in Tables 7 and 8.

Table 7: Predicted suspended sediment discharge and measured in (kg/s/m) for Niobrara River

Lane & Kalinske (1941) (Eq.(13))	Brook (1963) (Eq.(14))	Chang, S. & R, (1965) (Eq.(15))	Einstein (1950) (Eq.(11))	Bagnold (1966) (Eq.(12))	Van Rijn (2007) (Eq.(16))	q_{sw} measured
1111.548	4.9593	382.6514	0.1312	0.0459	5.99E-07	0.3403
93.29437	1.825	1.572097	0.0761	0.0488	4.67E-07	0.3443
133.9051	5.3309	6.150028	0.2896	0.1851	6.1E-06	1.0058
121.4647	4.5735	5.38059	0.3072	0.1957	6.22E-06	0.9588
47.95888	4.4843	4.257006	7.9223	0.4186	1.17E-05	1.3017
192.5111	1.9964	1.84872	0.1531	0.0563	7.14E-07	0.279
558.1278	2.9104	4.213199	0.1539	0.0401	4.61E-07	0.2441
1487.219	6.5146	16.60262	0.1528	0.0364	4.98E-07	0.3073
564.053	4.656	10.55829	0.2003	0.0468	7.94E-07	0.3379
152.3985	4.9486	4.343703	1.1622	0.1271	2.95E-06	0.8119

Table 8: Predicted suspended sediment discharge and measured in (kg/s/m) for Snake and Clearwater River

Lane & Kalinske (1941) (Eq.(13))	Brook (1963) (Eq.(14))	Chang, S. & R, (1965) (Eq.(15))	Einstein (1950) (Eq.(11))	Bagnold (1966) (Eq.(12))	Van Rijn (2007) (Eq.(16))	q_{sw} measured
53.72858	0.5429	0.71154	2.64304	1.9048	6.91E-05	0.1024
439191.8	24719	24262.9	0.23772	0.2554	8.62E-06	0.3331
511582.5	39086	28073	0.21755	0.2495	7.51E-06	0.4916
176538.9	30371	4842.18	0.05518	0.1531	5.28E-06	0.3581
51719.14	21778	1205.92	0.09904	0.1901	6.35E-06	0.1494
52.65163	0.7872	0.88327	20.8432	3.7626	0.00012	0.1747
44.02625	0.3582	0.54449	9.64382	4.0822	0.000117	0.0711
181.4077	1.3266	1.35342	8.94423	3.4495	0.000109	0.2086
112.0779	1.0842	1.42504	3.45271	3.8219	0.00012	0.2273
4419.628	4.1301	9.1845	5.68201	1.8549	6.55E-05	0.1641

Results of the statistical tests for suspended load equations are summarized in Table 9. The tests demonstrate that Bagnold, Einstein and Van Rijn gave the best predictions among other tested equations. Figures 12 to 21 show the comparison between computed and observed suspended loads.

Table 9: Summary of the results obtained from testing the accuracy of suspended load equations

No:	Name of river	Name of formula	MAE	RMSE
1	Mississippi	Einstein	2.135	3.115
2	Middle Loup	Einstein	0.174	0.221
3	Indian Canal data	Bagnold	1.004	1.536
4	Portugal	1- Bagnold	0.052	0.059
		2- Einstein	0.056	0.064
		3- Van Rijn	0.061	0.068
5	Niobrara	1- Bagnold	0.473	0.541
		2- Van Rijn	0.593	0.697
6	Rio Grande	Bagnold	1.873	2.212
7	Snake and Clearwater	1- Van Rijn	0.228	0.259
		2- Bagnold	1.849	2.420
8	Oak Creek	1- Bagnold	0.012	0.015
		2- Van Rijn	0.018	0.026
9	Colorado	1- Bagnold	0.588	0.756
		2- Einstein	0.588	0.788
10	Trinity	Einstein	0.203	0.279

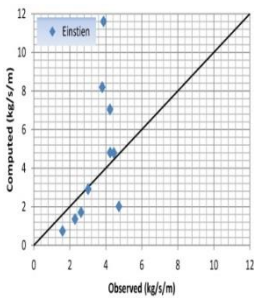


Figure 12. Graphical comparison between observed and computed suspended load for Mississippi River using Einstein (1950)

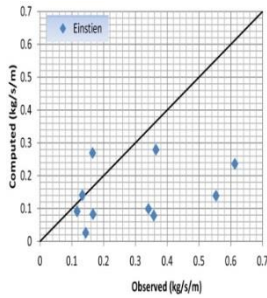


Figure 13. Graphical comparison between observed and computed suspended load for Middle Loup River using Einstein (1950)

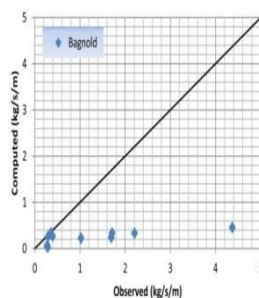


Figure 14. Graphical comparison between observed and computed suspended load for Indian canal data using Bagnold (1966)

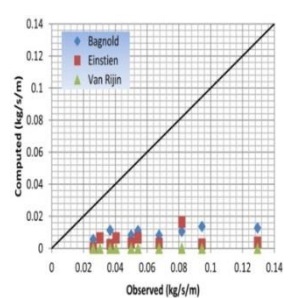


Figure 15. Graphical comparison between observed and computed suspended load for Portugal River using Bagnold, Einstein and Van Rijn equations

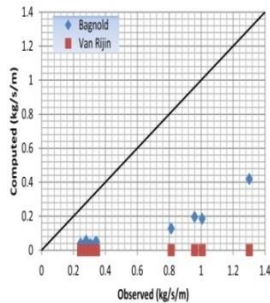


Figure 16. Graphical comparison between observed and computed suspended load for Niobrara River using Bagnold and Van Rijn equations

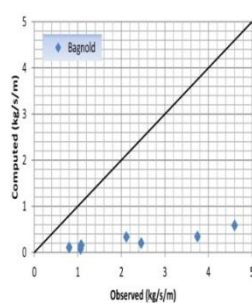


Figure 17. Graphical comparison between observed and computed suspended load for Rio Grande River using Bagnold (1966) equation

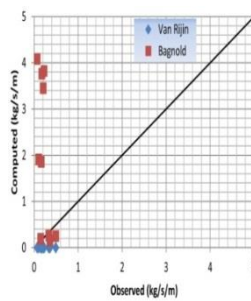


Figure 18. Graphical comparison between observed and computed suspended load for Snake and Clearwater River using Van Rijn and Bagnold equations

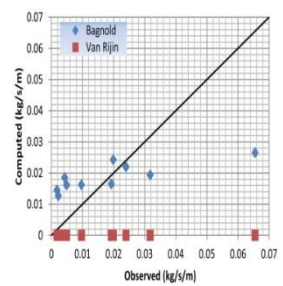


Figure 19. Graphical comparison between observed and computed suspended load for Oak Creek River using Bagnold and Van Rijn equations

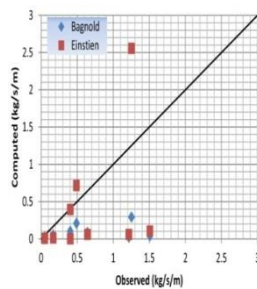


Figure 20. Graphical comparison between observed and computed suspended load for Colorado River using Bagnold and Einstein equations

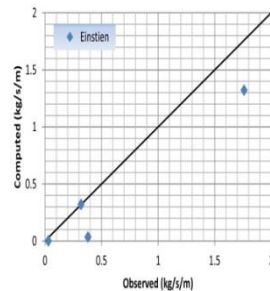


Figure 21. Graphical comparison between observed and computed suspended load for Trinity River using Einstein (1950) equation

Among the other tested equations, results demonstrated in Figures 12 to 21 and Table 9 confirm that Bagnold, Einstein and Van Rijn gave the least error in estimating the suspended load. The least values of Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) from testing Bagnold equation are equal 0.012 and 0.015 respectively. The Graphical comparison show that computed values of sediment discharge for rivers Mississippi, Oak Creek, and Snake and Clearwater are scattered around the line of perfect agreement while the majority of others gave under prediction compared with observed sediment discharge.

4.0 Conclusions

The accuracy of selected sediment transport equations have been tested using field data of 10 rivers around the world and the data describe the sediment, hydraulic and morphological characteristics of these rivers. Equations found with the most accurate sediment transport estimation are highlighted. For bed load estimation, validation shows that Einstein and Meyer-Peter Muller equations have least error compared with estimation obtained from other tested equations. These equations gave the best bed load estimation for three rivers while Schoklitsch equation and Du boys equation gave best bed load prediction for two rivers and one river respectively. The least values of Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) from testing Einstein equation using field data of Oak Creek River were found to be 0.02 and 0.04 respectively. For estimation of suspended load, Bagnold, Einstein and Van Rijn gave the least error compared with the results obtained from applying other tested equations. The least values of Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) obtained from testing Bagnold equation are found to be 0.012 and 0.015 respectively. Validation of the selected sediment transport equations show that there is no unique equation that can always give accurate prediction for all rivers. This is can be attributed to the fact that different rivers has different hydraulic and morphological characteristics such as discharge, velocity, energy slope, bed forms, median diameter, and sinuosity.

Notations

$q_{b,w}$ = Bed load transport (Kg/s /m)

ϕ = Einstein bed load function

s = slope

ν = viscosity of the fluid

ρ_s = sediment density (kg /m³)

γ_s = specific gravity of sediment ($\rho_s * g$)

ρ = fluid density (kg /m³)

g = gravity acceleration (m/s²)

D_{50} = particle diameter (m)

$\frac{P}{B} = \tau V = \rho g R s V$

V = mean velocity m/s

e_b = efficiency factor of bed load

$\tan \alpha$ = coefficient obtained

$$k_3 = \frac{0.173}{D_s^{3/4}}, q_{b,v} = \text{bed load transport rate (m}^3/\text{s/m)}$$

q = discharge per unit width (m³/s/m)

q_w = discharge in unit of (kg/s/m)

$$T_c = 0.12 (\gamma_s - \gamma) D$$

$$k = \text{Strickler roughness equation} = 1/n = \frac{V}{R^{2/3} S^{1/2}}$$

$$k' = \text{roughness coefficient due to the bedforms} = \frac{26}{D_{90}^{1/6}}$$

$$q_c = 0.26 \left[\frac{\gamma_s - \gamma}{\gamma} \right]^{5/3} \frac{D^{3/2}}{S^{7/6}} \text{ in unit (m}^3/\text{s/m)}$$

$$M_e = \text{mobility parameter} = \frac{(V - u_{cr})}{[(S-1)gD_{50}]^{0.5}}$$

d = water depth

u_{cr} = critical velocity

$$u_{cr} = 0.19(D_{50})^{0.1} \log \left[\frac{12 d}{3 D_{90}} \right] \text{ for } 0.0001 < D_{90} < 0.0005 \text{ m}$$

$$u_{cr} = 8.5(D_{50})^{0.6} \log \left[\frac{12 d}{3 D_{90}} \right] \text{ for } 0.0005 < D_{90} < 0.002 \text{ m}$$

$$\Phi = 13 * \Omega * \text{EXP} \left[\frac{-0.05}{\Omega} \right]$$

$$\Omega = \frac{\tau * U_*}{\rho [(S-1)gD_{50}]^{3/2}}$$

$q_{s,w}$ = Suspended load transport (kg/s/m)

$$C_a = \text{reference concentration (volume)} = \frac{1}{11.6} \frac{q_{b,w}}{u_* a}$$

a = reference level = $2D_{65}$

$$Pe = 2.303 \log \frac{30.2 d}{\Delta}$$

$A = a/h$ dimensionless reference level

$Z = w_s / (k u_*')$ suspension number, the I_1 and I_2 integrals can be determined graphically relate to the A and Z

ω = fall velocity of sediment (m/s)

e_s = efficiency factor of suspended load

C_a = concentration by weight at $y = a$

P_L = factor in a function of $\frac{\omega}{U^*}$ and $\frac{n}{d^6}$

C_{md} = reference sediment concentration at $d/2$ where d is the depth of flow

k = Von Karman constant = 0.4, $Z_1 = \frac{Z}{\beta}$, $Z = \frac{\omega}{k U^*}$

I_1 and I_2 determined from the graph in term of ξ_a and Z_2

$\xi_a = \frac{a}{d}$

$Z_2 = \frac{2 \omega}{\beta U^* k}$, D^* = dimensionless particle size.

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