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HIGH-CYCLE FATIGUE IN CONCRETE THROUGH THE THEORY OF CRITICAL DISTANCES: FROM PERSPECTIVE OF WATER-CEMENT RATIO

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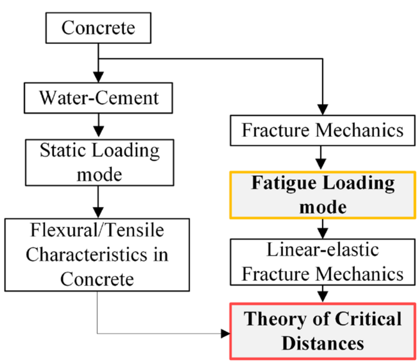
Graphical abstract



Abstract

Water-cement ratio plays a unique role in concrete structures. The uniqueness of evaluating concrete from the perspective of the water-cement ratio will be more obvious if the structure is being assessed down into its microscale level. It is important to realise that most dynamic concrete structures are hydro-related structures, and those structures need to be designed as detail as possible. Thus, the design of dynamic concrete structures has to incorporate accurate fatigue formulation and precise water-cement ratio variation effect. Currently, one of the most simplified yet accurate formulations proposed to run fatigue cases throughout a wide spectrum of scope is the Theory of Critical Distances (TCD). Therefore, the article reviews and discusses the precision of TCD towards the water-cement ratio perspective.

Keywords: concrete, water-cement ratio, high-cycle fatigue, fatigue and fracture mechanics

1.0 INTRODUCTION

Water-cement ratio should not be literally explained as the amount of water in terms of mass to the amount of cement in a concrete mix. Water-cement ratio must be understood down to the microstructure scale. The relationship to the spacing between cement particles, the period for cement in its hydration process, and to be more detail, the changes that different water-cement ratio in concrete could cause to the pore surface tension, capillary stress so on and so forth which all of them weakens the concrete structure (Bentz and Aïtcin, 2008).

From the construction industry perspective, water-cement ratio appears as moisture in concrete mix. Most of the dynamics concrete structures deal with the extreme state like dams, bridge piers, harbours and ports. All of the mentioned concrete structures encounter continuous changes in tides. Thus, the concrete structures are eventually under moisture or can be expressed as water-cement ratio variation at all times (Bartlett and MacGregor, 1994). Direct and typical influence in the change of water-cement ratio to the concrete's compressive strength is one of the basic engineering design parameters but one should not abandon the fact that other characteristics in concrete such as flexural or tensile strength do play a significant role in a concrete structure.

On top of that, Petersson, who is one of the pioneers in exploring fracture mechanics in a brittle material, concluded in his research mentioning that water-cement ratio plays an important role in shifting fracture characteristics in concrete as it affects the type and size of porosity and eventually lead to changing interfacial transition zone (ITZ) and fracture process zone (FPZ) (Petersson, 1980). Hence, the exact effect of water-cement ratio variation in concrete mix towards concrete's mechanical properties is significant (Chen, Huang and Zhou, 2012).

2.0 FATIGUE IN CONCRETE

In calculating critical distance *L* in the Theory of Critical Distances (TCD), two parameters govern the formulation – fatigue crack growth threshold ΔK_{th} and plain-specimen fatigue limit $\sigma_{0,max}$.

Here's the fascinating part of TCD. The value of *L* can be obtained from the intersection of linear-elastic stress field at the crack tip vicinity and the material's un-notched endurance limit (Susmel and Taylor, 2010). In detail, based on the Theory of Critical Distance (TCD) using Point Method (PM), failure will occur, when linear elastic stress σ_{xx} at a distance $L/2$ from the notch tip is equal to the material's maximum un-notched endurance limit $\sigma_{0,max}$. Thus, at the failure point which is considered the critical state of a material, each parameter in the formulation will be in its critical condition as well. For concrete under fatigue manner, (i) fatigue crack growth threshold ΔK_{th} is the point where concrete could start propagating a crack; and (ii) endurance limit $\sigma_{0,max}$ which denotes maximum stress magnitude that concrete can endure at finite number of cycles (Boyer, 1986). Beyond ΔK_{th}, the concrete structure is insecure because the crack will start propagating until it reaches the fracture point which is concrete's plain-strain fracture toughness $K_{Ic,concrete}$ (Dahlberg and Ekberg, 2011). The concept is known as Paris Law.

2.1 Fatigue Crack Growth Threshold

It is tedious and difficult to obtain fatigue crack growth threshold ΔK_{th} and the outcomes are not guaranteed (Santus, Taylor and Benedetti, 2018). Before plotting the Paris Diagram, data and plotting on crack size, *a* against number of cycles, *N* has to be acquired first (Figure 1). The slopes of the graph will later be presented in Y-axis for Paris Diagram. Paris Law diagram consists three phases (Figure 2) – basically Phase 1 which is approximately below ΔK_{th} where crack will not propagate, Phase 2 where the crack obeys Paris power law and finally Phase 3 where the crack in specimen or structure will accelerate and fracture. However, fatigue crack growth threshold ΔK_{th} is insensitive to short crack which that are what naturally will happen on concrete when it fails; usually short cracks will appear (Taylor, 2004). Concrete structure does not need long cracks to justify whether it has failed or not – it is suffice to consider the structure has failed when it can no longer bear its own ultimate loading (Neville, 2011).

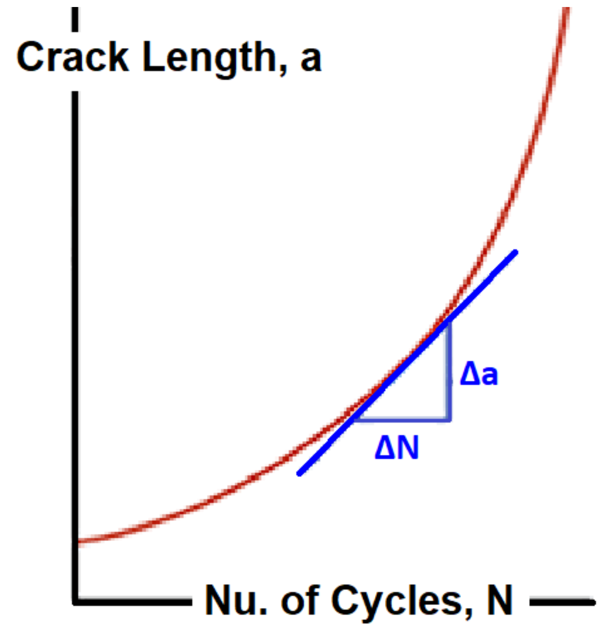


Figure 1 Crack Length against Number of Cycle plot

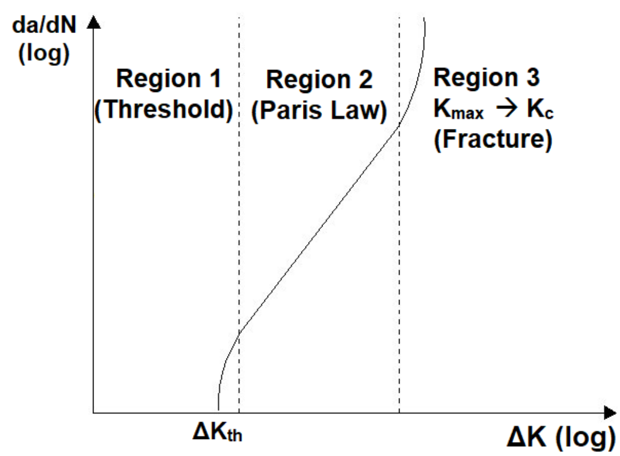


Figure 2 Typical fracture mechanics fatigue crack propagation behaviour (Kamble, Raykar and Jadhav, 2020)

Consequently, due to its incompatibility to the concrete's crack pattern and difficulties to run the experiment and achieve consistent outputs, eventually, the Theory of Critical Distance (TCD) provides huge restitution without jeopardizing the result's precision. As mentioned above, the correspondence between the linear-elastic stress field at the notch tip σ_{xx} and the endurance limit of a material $\sigma_{0,max}$ will equal half of the critical distance magnitude, $L/2$ as in Figure 3.

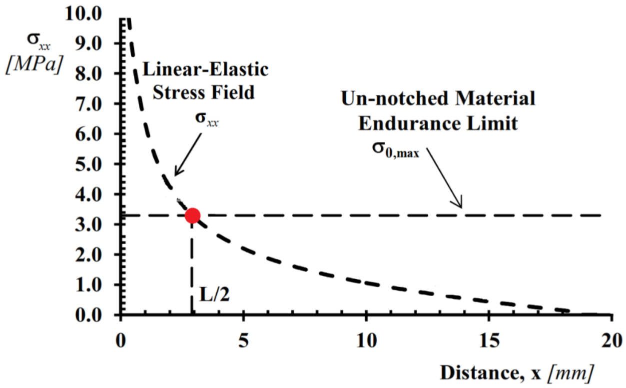


Figure 3 Linear-elastic stress fields in the endurance limit condition and accuracy of the PM in estimating fatigue strength of the tested concrete (Susmel, 2016).

2.2 S-N Curve

The un-notched endurance limit of a material $\sigma_{0,max}$ can be obtained through the *S-N* curve graph. Y-axis of the graph is the maximum amplitudes of stress exerted σ_0 for certain numbers of cycle to failure, N_f. The experiment can be conducted by testing concrete beams under a three-point bend basis configuration. The concrete beams are applied under a flexural loading manner understanding that flexural strength is also acknowledged as the tensile strength of concrete (Onwuka, Temitope and Awodiji, 2015). Figure 4 is an typical *S-N* curve plotted. To achieve better curvature of the graph, more specimens are needed to test until it fails.

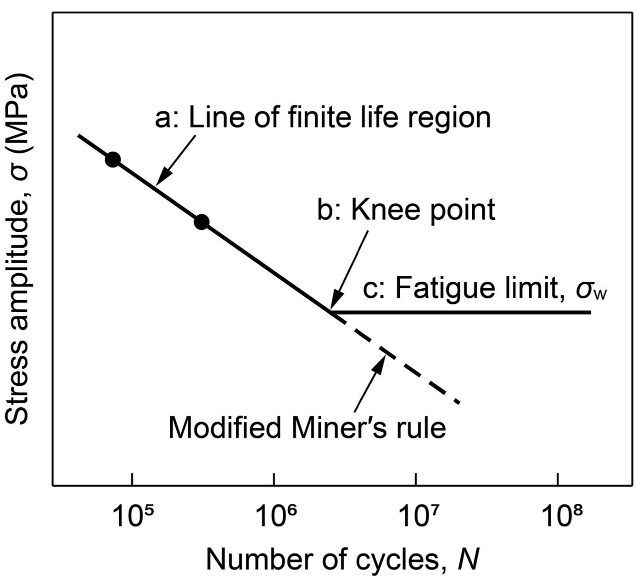


Figure 4 Composition of S-N curve: Line of finite life regime, knee point N_{knee} and fatigue limit σ_w (Murakami *et al.*, 2021)

The stresses were chosen using few percentages of its ultimate flexural strength. The range of percentages decided based on ACI 215R-74 were 20 to 80% from concrete's ultimate strength σ_u (ACI Committee 215, 1992).

2.3 Linear-Elastic Fracture Mechanics

The fundamental assumptions are the same as those in features of materials: the material is a homogeneous isotropic continuum (that is microscopic inconsistencies in a structure are ignored); stress is relative to strain; strains are trivial; and distortions are neglected. The material is anticipated non-self-equilibrating inner stresses. Several alterations can be made to the basic theory to take into consideration the real performance of the materials (Pook, 1972).

LEFM might display a better relationship between the manufactured construction materials (which usually contain irregularities) and laboratory specimens (which often be ideal without flaw). LEFM is predominantly controlled by the region of crack tip yielding and the amount of stress exerted (Irwin, 1957). LEFM is violated if one of these two control variables is not followed. It is easier to understand by referring to the Kitagawa-Takahashi in Figure 5.

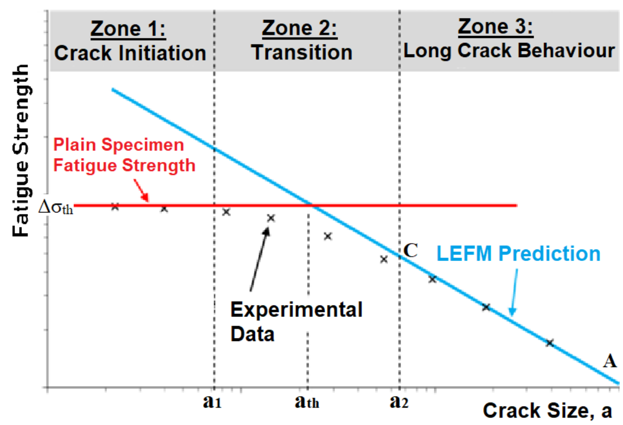


Figure 5 Schematic representations of Kitagawa and Takahashi's diagram (Kitagawa and Takahashi, 1976; Bellett, Pessard and Morel, 2014)

Based on Kitagawa and Takahashi's diagram above, we can see the effective region of LEFM (A-C-a_2) is restricted by material plain fatigue limit or threshold fatigue limit line, $\Delta\sigma_{th}$ and a_2 defines the crack size at which small crack effects end (Susmel and Taylor, 2011). The formalization used when the crack is less than a_2 or stress range more than $\Delta\sigma_{th}$ is no longer governed by LEFM. Going beyond the fatigue stress limit such $\Delta\sigma_{th}$ might violate LEFM because it is considered to be an excessive plasticity material (Kitagawa and Takahashi, 1976), hence the operational concept for it is different as explained previously. So, in the study, the disparity between crack length and fatigue limit stress must be well-understood.

2.5 The Theory of Critical Distances

To apply the Theory of Critical Distances (TCD), two regulations must be complied with, so that the subsequent concept of TCD will be operational. The first one is TCD assumes that the material being examined is at the endurance limit provided the range of effective stress calculated using TCD, $\Delta\sigma_{eff}$ is less than the range of un-notched endurance limit, $\sigma_{0,max}$ (Susmel, 2016). The second material property is TCD has to consume critical distance L based on the following formula as in Equation 1 (Taylor, 2006).

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\sigma_{0,max}}\right)^2 \quad (1)$$

3.0 THE SIGNIFICANCE OF THE WATER-CEMENT RATIO TO THE THEORY OF CRITICAL DISTANCES

Lowering concrete's water-cement ratio will result higher overall concrete compressive strength (Neville and Brooks, 2010; Kosmatka, Kerkhoff and Panarese, 2011; Wang *et al.*, 2020) When the matrix bond is stronger, the strength of cement paste is equal to the aggregate strength – and if cracks propagate, it will cut through the cement paste and also aggregate. Contrariwise, if the matrix bonds are weaker, cement's strength is less than aggregate – and when it cracks, the crack will flow through cement but will deflect when there are aggregates.

In metal, research has found that microstructure inhomogeneity affects the fatigue behaviour of the material (Liu *et al.*, 2017). But there is none of the fatigue characterisations in concrete. The hypothesis was upraised to determine if the Theory of Critical Distance (TCD) is only accurate at a certain level of water-cement ratio due to the physical properties of the microstructure.

The study postulates as such because referring to the previous study (Susmel, 2016); the Theory of Critical Distance (TCD) gave an indefinitely small error in testing fatigue strength of notched concrete with a water-cement ratio of 0.4. Meanwhile, when the test is on concrete with a slightly higher water-cement ratio (which is 0.5), the error became more substantial – and it cannot be left trivial. Hence, is the Theory of Critical Distance (TCD) only effective on a certain level of water-cement ratio for concrete?

From material analysis, concrete with a water-cement ratio of 0.3, 0.4 and 0.5 are in a different class of concrete (British Standards Institution (BSI), 2013). This also means all batches will have distinctively different properties – might be in terms of physical and chemical properties (Kosmatka, Kerkhoff and Panarese, 2011).

As we can see from Table 1, concrete with a water-cement ratio of 0.3 is classified as very-high strength concrete, concrete with a water-cement ratio of 0.4 is as high-strength concrete and concrete with a water-cement ratio of 0.5 as conventional concrete. The postulation of crack propagation comes into effect when the study compares the strength of aggregate and concrete. The strength of aggregate will be at least about 65 MPa (Kosmatka, Kerkhoff and Panarese, 2011).

Perhaps due to these factors, the error given out by Theory of Critical Distance (TCD) variate, which TCD did not incur into its formulation. Thus, do varying water-cement ratios affects significantly the precision of TCD?

Table 1 Strength Classification of Concrete (Farny and Panarese, 1994)

Parameter	Conventional concrete	High-strength concrete	Very-high strength concrete	Ultra-high strength concrete
Strength, MPa	< 50	50–100	100-150	> 150
Water-Cement Ratio	> 0.45	0.45 – 0.30	0.30 – 0.25	< 0.25
Chemical Admixtures	Not necessary	WRA/HRWR*	HRWR*	HRWR*
Mineral Admixtures	Not necessary	Fly Ash	Silica Fume**	Silica Fume**
Permeability Coefficient (cm/s)	$> 10^{-10}$	10^{-11}	10^{-12}	$< 10^{-13}$
Freeze-thaw Protection	Needs air entrainment	Needs air entrainment	Needs air entrainment ^{x}	No freezable water ^{x}

*WRA = Water Reducing Admixture; HRWR = High Range Water Reducer

**Also may contain fly ash; ^{x} Porosity, Freeze-Thaw Durability, and Corrosion Resistance

4.0 CONCLUSIONS

In order to evaluate the TCD sensitivity and precision towards water-cement ratio variation in concrete, it is wise to refer to the fundamental concept of both the water-cement ratio and TCD itself. One should appreciate that changing a factor such as the water-cement ratio will change the mechanical properties of concrete (Singh, Munjal and Thammishetti, 2015), and the critical distance, L is one of the mechanical properties in formulating TCD. Therefore, it will definitely change and affect the TCD as well.

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Declaration

The authors have no conflicts of interest to disclose. They have no known competing financial interests or personal relationships that could have influenced the work reported in this research article.

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