

# A HYBRID APPROACH TO TRAFFIC BREAKDOWN DETECTION USING DENSITY PERCENTILE ANALYSIS AND FUZZY LOGIC CONTROL

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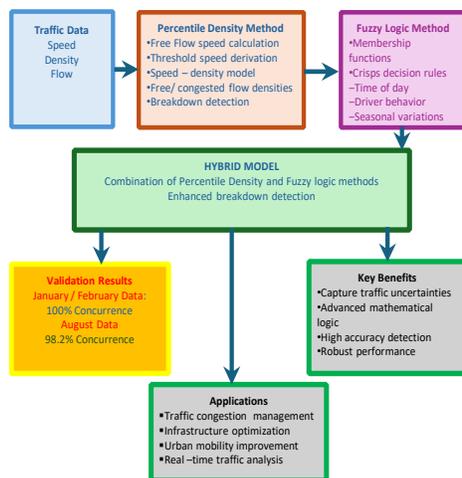
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## Graphical abstract



## Abstract

This study produces a novel hybrid model to predict traffic flow breakdowns. The procedure started with the creation of a Percentile Density Approach (PDA) which identified traffic flow breakdown using a percentile -based analysis of density. In this method, traffic flow breakdown was found between traffic free-flow and congested states. Afterwards, a fuzzy logic approach was developed to advance the process into a model form whilst incorporating other factors. As a result, this model integrates the major parameters in the PDA, such as free-flow and congested density as well as weather conditions, driver behavior, and seasonal factors. Thus, the fuzzy hybrid logic system developed, uses membership functions built around both the PDA and these additional parameters enhanced by well-defined sets of crisps decision rules. This resulted in a model which describes traffic flow breakdown incorporating fundamental traffic variables with real-world uncertainties.

**Keywords:** free-flow speed, speed drop, traffic breakdown, fuzzy logic control, percentile density analysis

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## 1.0 INTRODUCTION

Traffic breakdown refers to the transition of traffic states from free-flow to congested traffic conditions. This phenomenon is a major cause of urban traffic problems, resulting in significant environmental and economic consequences. Despite decades of research, accurately predicting the onset of traffic breakdown remains a persistent challenge. Traditional threshold-based models often fall short in capturing the complex and dynamic nature of traffic transitions, especially under the

influence of diverse factors such as driver behaviour, weather conditions, and infrastructure variability [1,2]. These limitations hinder traffic management and delay timely interventions, ultimately compromising the attainment of sustainable transport.

Existing methodologies typically rely on theoretical speed density relationships to establish distinctions between “free flow” and “congested” traffic states [3,4]. However, real-world traffic data often exhibit skewed patterns influenced by drivers’ behavior and percentile – based approaches have been introduced to better accommodate this skewness [5–9].

Nonetheless, these methods still require refinement to handle driver heterogeneity, seasonal flow variations, the growing instability of traffic flow particularly when applied to traffic breakdown detection [10–13].

Fuzzy logic known for its ability to handle uncertainty and imprecision, has shown promise in traffic signals and congestion modeling [1,14–17]. However, its application in modeling traffic breakdown remains limited, despite its potential to incorporate vague and overlapping conditions such as driver behaviour and environmental influence. The lack of behavioural and contextual considerations in conventional models often results in reactive rather than predictive responses to breakdown events.

Fuzzy logic with membership functions and crisps decision rules can be applied to traffic breakdown in the following manner: if membership functions are minimum then one condition for breakdown is met [18,19]. This is because  $\mu_{HD}(k_c) = 0$  can lead to a collapse in kinematic waves, known as stop-and-go waves that causes traffic breakdown.  $\mu_{HD}(k_f) = 0$  is an indication of erratic lane-changing that leads to turbulence in traffic which can also cause traffic breakdown.  $\mu_{HD}(C_t) = 0$  implies headway inconsistency that can cause shockwaves to propagate in traffic [20–22]. The membership of the percentile congested density  $\mu_{HD} K_c(x)$  reaching a value of 1 or exceeding it captures headway consistency thresholds that triggers breakdown since it represents a critical transition in traffic flow stability as emphasized in the three- phase traffic theory [23,24]. The membership of a fuzzy logic can also be formulated to capture daily and seasonal variation in traffic conditions [25,26]. This condition means that at low flows, breakdown requires perfect headway consistency [27,28] which are the result of rigid vehicle following that causes reduced reaction margin. It could also be caused by sudden brake by drivers. High flow breakdowns which is indicated by  $k_f > 50\%$  are usually found during the morning rush hours of between 06:00-09:00.  $\mu_{HD}(C_t) > 1$  which occurs in the daytime can be cause by headway degradation [29].

This study therefore bridges this gap by formulating a hybrid model that brings together percentile density analysis with a Binary Fuzzy Logic Controller (BFCL). Using trapezoidal membership functions, the model identifies transitional states between free-flow and congestion based on percentile density thresholds, effectively considering the skewness of traffic distributions. It also accounts for traffic breakdown triggers such as erratic lane changes, shockwaves and temporal traffic variations. This model is validated under varying weather conditions (dry and rainy seasons) along the highly congested Keffi- Abuja motorway at Masaka, Nigeria.

Through this approach, the study contributes to advanced real- time management of traffic through a robust real-time traffic detection of traffic states and traffic breakdown, enhancing the responsiveness and sustainability of traffic management strategies.

The remainder of this paper is structured as follows: section 2 outlines the data collection methodology, section 3 details the hybrid modeling approach, the findings and discusses their implications, and section 4 concludes the study.

## 2.0 METHODOLOGY

### 2.1 Data Collection

Masaka, located on the busy Keffi-Abuja corridor in Nasarawa State, Nigeria, served as the study sites. The area features a roadside market adjoining the highway, which contributes significantly to persistent traffic congestion for commuters traveling toward Abuja. The study focused on a section of the A234 trunk road, a major north-south corridor within Nigeria's administrative road network, with speed limit 80km/h. This segment comprises a three-lane dual carriageway with a total width of 9 meters and separated by a 1.2-meter-high concrete median barrier. Paved shoulders, each approximately 1.5 meters wide, are present on both the median and outer sides of each carriageway. The geographic coordinates of the survey location is 9°00'07.15" N, 40°27'00.0" E and 9°00'22.3" N, 40°27'02.2" E."

A high-resolution video camera mounted on a pedestrian overpass was used to record vehicle speeds, classifications and volumes. The monitored road segment is 900m long and carries heterogenous traffic, including tricycles, motorcycles, cars, vans, buses and trucks and trailers. These vehicles were categorized according to the EU vehicle classification systems: L (tricycles and motorcycles), M (cars and minivans), N (goods vehicles), and O (trucks and trailers) [30–33], providing a consistent basis for density analysis.

Data collection was conducted on weekdays (Monday to Thursday) over two weeks in January and February 2023, and one week in August 2023. The data collection periods were selected carefully to capture peak and non-peak traffic activity, enable seasonal comparison and provide preliminary validation. Specifically, data were collected from 9-12<sup>th</sup> January, 16-19<sup>th</sup> January and 6-9<sup>th</sup> February, and 20-23<sup>rd</sup> February, representing the dry season, and from 8–11<sup>th</sup> August), representing the rainy season. The August data served as a validation period to test whether the patterns and relationships observed in the January–February models remained consistent under different seasonal and travel conditions. All data collection was conducted from 6:00AM to 6:00 PM throughout all five survey weeks, yielding a total of 33,465 speed data points for January and February, and 8,416 speed data points for August. The location of the Masaka data collection site is illustrated in Figure 1.

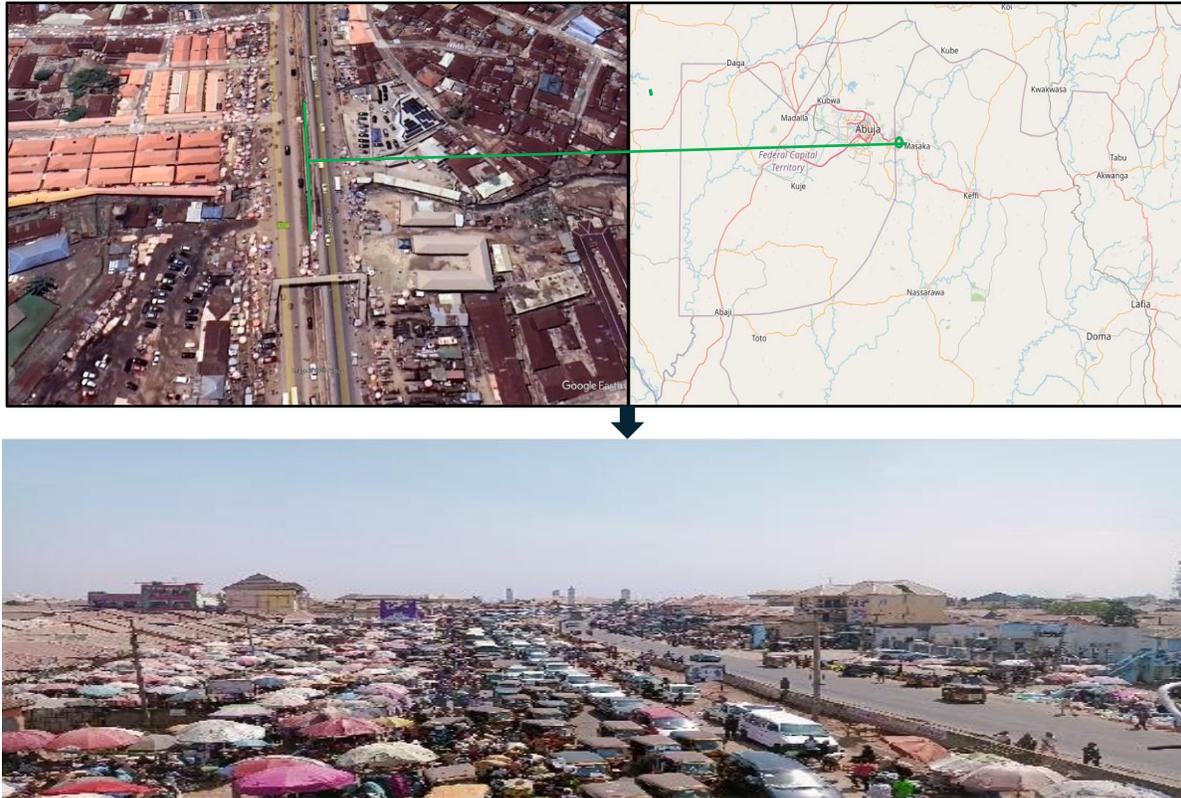


Figure 1 Map and picture of study location

## 2.2 Data Processing

In this study, the free flow speed was determined based on the highest observed prevailing speed. This is followed by the determination of speed drop, threshold speed and then the classification of traffic states as congested or uncongested. The speed density model for the motorway was developed to determine percentile densities under free-flow and congested conditions, and to identify the breakdown state of the road.

## 2.3 Data Analysis: Percentile Density Methodology for breakdown detection

In this study, the prevailing free flow speed and the threshold speed were first derived. Following the camera captured observations of speed and volume, traffic flow was derived and then the speed-density model was developed. The prevailing free-flow speed was substituted into the speed-density model to derive the corresponding free-flow density ( $K_{ff}$ ). Also, the minute-by-minute densities of individual vehicles ( $K_b$ ) was derived from the speed-density model. Furthermore, the threshold speed was used in the speed density equation to derive the congested density at every minute ( $K_{cs}$ ). Finally, the ( $K_b$ ) was compared with the ( $K_{ff}$ ) and ( $K_{cs}$ ) to derive the breakdown state of the road on a percentile basis to occur when ( $K_b$ ) lies between ( $K_{cs}$ ) and ( $K_{ff}$ ). At every minute, the number of vehicle densities  $K_b$  calculated between the free-flow density  $K_{ff}$  and the congested density  $K_{cs}$  was identified to represent the breakdown of the road. The proportion of these instances relative to non-breakdown states was calculated to determine

the percentile breakdown percentage. This breakdown scenario occurs due to limited roadway capacity at high traffic flow conditions, triggering sudden reductions in speed and significant congestion. The percentage of breakdown exceeding 50% indicates higher probability of breakdown and was used as the breakdown of the road.

To create input dataset for the membership function and crisps decision rules for the hybrid percentile fuzzy approach, density of traffic flow derived in section 2.3, that is, ( $K_{ff}$ ), and ( $K_{cs}$ ) were expressed as percentiles in each minute. Two distinct percentile densities represented as  $k_f$  and  $k_c$  were thus created and used as described in the next subsection.

### 2.3.1 Binary Fuzzy Logic Controller (BFLC): Hybrid Percentile -Fuzzy approach

The procedure consists of fuzzification in which percentile densities of free flow  $k_f$ , congested flow  $k_c$  and congested traffic state  $C_t$  were mapped to a trapezoidal membership function in order to identify the gradual transition to breakdown. The trapezoidal membership function is used to translate gradual traffic state transitions. This is followed by a Crisps decision rule procedure where binary logic rules were applied.

The final breakdown takes into account this first condition along with other conditions such as the percentile free flow density rising above or below 50%. Other conditions considered include the membership of the percentile congested density  $\mu_{HD} K_c(x)$  reaching a value of 1 or exceeding it. Other membership conditions considered include morning (6:00

am to 9:00am) period as well as daytime to evening (9:00am to 6:00pm) in order to derive seasonal traffic variation based on free flow density variations [25,26]. Thus, the BFLC model presented in this paper captures both daily, seasonal as well as drivers' behavior.

Finally, a validation was performed, in which case, the BFLC's output was cross verified against the percentile density method, achieving a >98% agreement for January/ February and August 2023. The Figure 2 shows the flowchart for the BFLC.

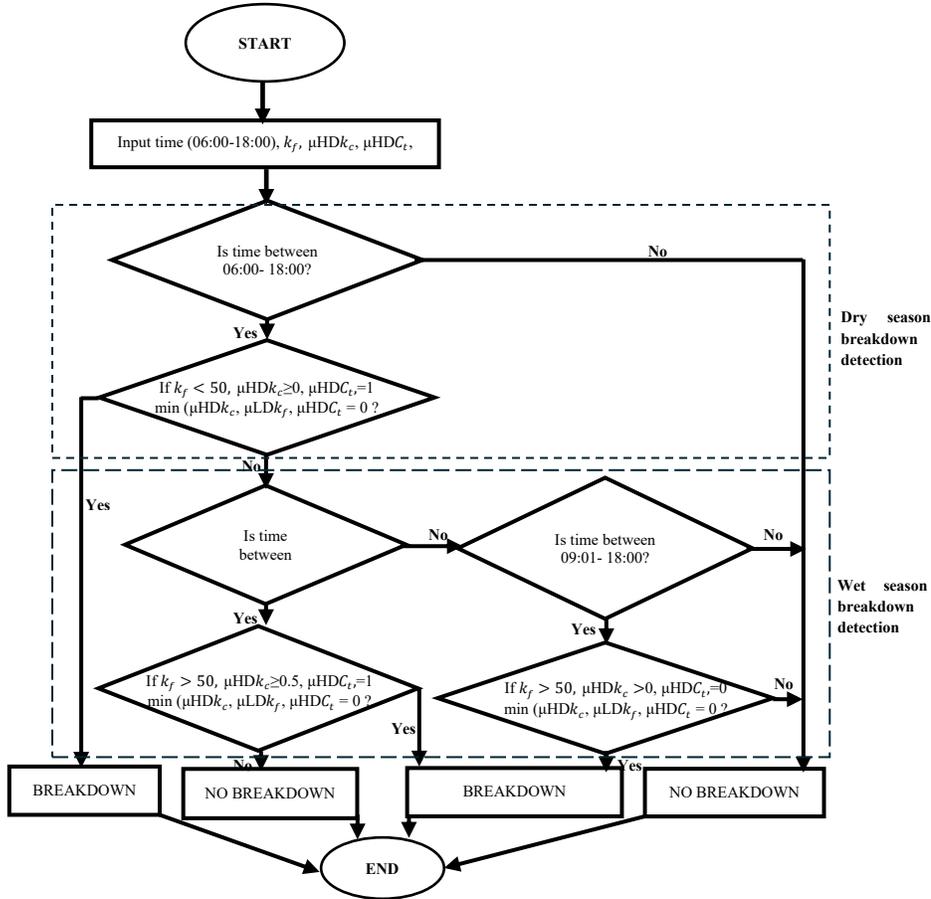


Figure 2 breakdown model flowchart

The flowchart in Figure 2 shows that the model is only active for time within the 6:00 to 18:00, and breakdown fails outside this range of time as off-peak/night traffic rarely experience critical density. The initial trigger of the breakdown occurs when  $min [\mu HD(k_c), \mu LD(k_f), \mu HD(C_t)] = 0$ . If these conditions are met, the flowchart further checks low-flow breakdowns as indicated by  $k_f=50\%$  [34], these low flow conditions are not common during the morning rush hours especially on the Keffi-Abuja route where commuters troop early towards the city center in the morning. If the low – flow condition is met, the flowchart further checks the condition  $\mu HD(C_t) = 1$  that incorporates lane changing behavior to the model.

### 3.0 ANALYTICAL PROCEDURE VALIDATION AND DISCUSSION

#### 3.1 Traffic Breakdown from Percentile Density Analysis

The procedure for the percentile density approach started with estimating the free-flow speed, derived as the highest recorded speed, followed by the calculation of the threshold speed. Using the traffic data of speed and flow obtained from the camera, the speed density model was derived as presented in the Table 1. The formulated speed–density model was used to derive traffic densities of both free-flow and congested conditions and calculate minute-by-minute vehicle densities. These values were the basis for determining the breakdown status of the motorway. The following subsections detail the procedure used to identify breakdown conditions in the percentile density analysis framework.

Table 1 Linear Regression coefficients results for speed density

| Speed-Density model for January to February 2023 |                             |                | Speed-Density model for August 2023 |                             |                |
|--|-----------------------------|----------------|-------------------------------------|-----------------------------|----------------|
| Model  | Equation                    | R <sup>2</sup> | Model                               | Equation                    | R <sup>2</sup> |
| Linear Fit                                       | $v = -0.1637k + 81.524$     | 0.91           | Linear Fit                          | $v = -0.1591k + 82.185$     | 0.74           |
| Log Fit  | $v = -45.09\ln(k) + 289.07$ | 0.91           | Log Fit                             | $v = -48.91\ln(k) + 312.79$ | 0.74           |
| Exponential Fit                                  | $v = 125.96e^{-0.005k}$     | 0.91           | Exponential Fit                     | $v = 154.76e^{-0.005k}$     | 0.73           |
| Power fit  | $v = 41662k^{-1.262}$       | 0.91           | Power fit                           | $v = 246209k^{-1.566}$      | 0.71           |

The speed-density data were fitted to linear, logarithmic, exponential, and power models to determine the best-fitting model. The R-squared showing the fit of the model with their equations is shown in the Table 1.  $v$  represents speed, while  $k$  is density. The Table 1 shows that the R-squared for January–

February for all the models are 91%. For August 2023, the R-square for Linear and Logarithm showed better fit with 74%. Therefore, Linear model was selected as our speed density relationship. The plot of the speed density graphs for January – February and August 2023 is as shown in the Figure 3.

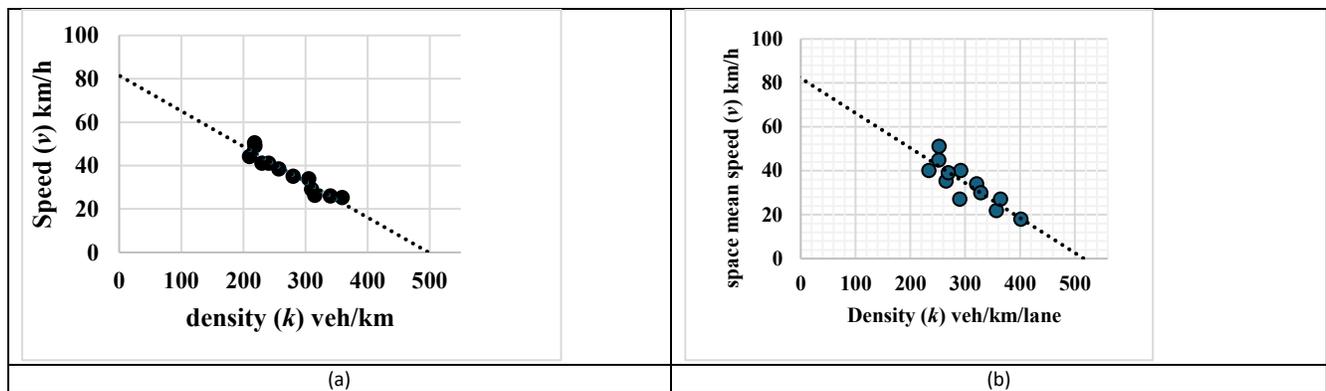


Figure 3 Linear speed density models of (a) January – February, (b) August

3.2 Residual analysis

The speed – density model was verified using residual analysis

on the January-February data and the August data as presented on Figure 4

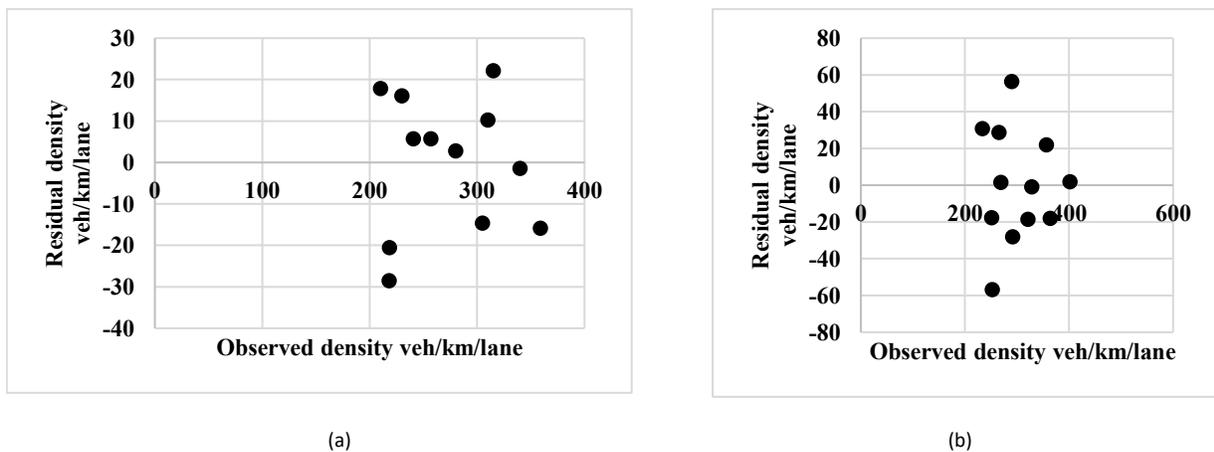


Figure 4 Residual plot to validate speed density equation for weekdays of (a) January and February, (b) August

The residual plot in Figure 4 showed that the data is randomly scattered around the horizontal axis which implies that the residuals are randomly distributed indicating the model describes the real data validating the January-February as well as August speed density equations.

After deriving the speed- density, the density at free and congested traffic speed were obtained to derive the breakdown status of the road as explained in subsection 3.3.

### 3.3 Breakdown from percentile density Analysis

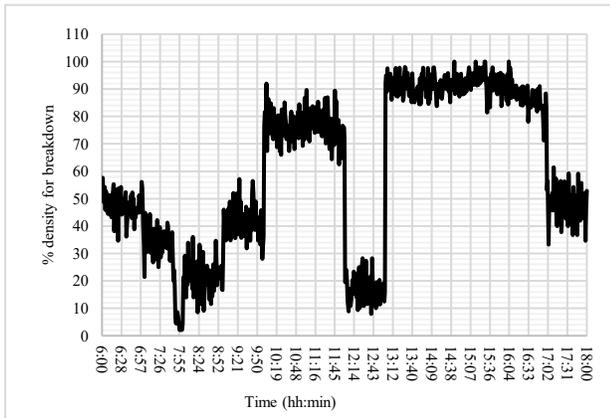
To drive these percentiles, the free flow speed and the congested speed for weekdays were used as shown in the Table 2

Table 2 weekdays density at congested and free flow states

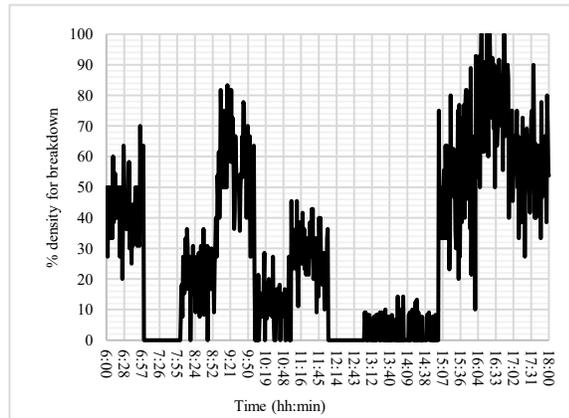
| Period           | Speed                       | Density  |
|------------------|-----------------------------|--|
| January-February | Free-flow speed = 49.8 km/h | Density at free flow speed, $K_{ff}$<br>$= \frac{(81.524-49.8196)}{0.1637}$<br>= 193.7 veh/km  |
|                  | Congested speed =30.1 km/h  | Density at congested speed, $K_{cs}$<br>$= \frac{(81.524-30.13319)}{0.1637}$<br>= 313.9 veh/km |
| August           | Free flow speed = 50.8 km/h | Density at free flow speed, $K_{ff}$<br>$= \frac{(82.185-50.8297)}{0.1591}$<br>= 197.1 veh/km  |
|                  | Congested speed =38.4 km/h  | Density at congested speed, $K_{cs}$<br>$= \frac{(82.185-38.4394)}{0.1591}$<br>= 275.0 veh/km  |

Table 2 for each minute of the data , the densities at every minute ( $K_b$ ) that lies between  $[K]_{ff}$  and  $K_{cs}$  were calculated and

their percentage against no breakdown was calculated and presented in the Figure 5 [35].



(a)



(b)

Figure 5 Weekdays percentile density for breakdown of (a) January – February, (b) August

Traffic flow breakdown indicated in Figure 5 were identified when the percentile density between the congested and free flow ( $PDCF$ ) exceeded 50%, during the January-February period.

To run preliminary check on the breakdown derived in this procedure, we derived the congestion of the road as speeds that are below the threshold speed [8] and compared the times, they occurred with the times the breakdown was detected in the

percentile density procedure. The congestion marker was determined when the percentage of congested state exceeds 50%. As presented in Figure 6, empirical results demonstrates that traffic flow breakdown do not necessary happen with the onset of congestion [2].

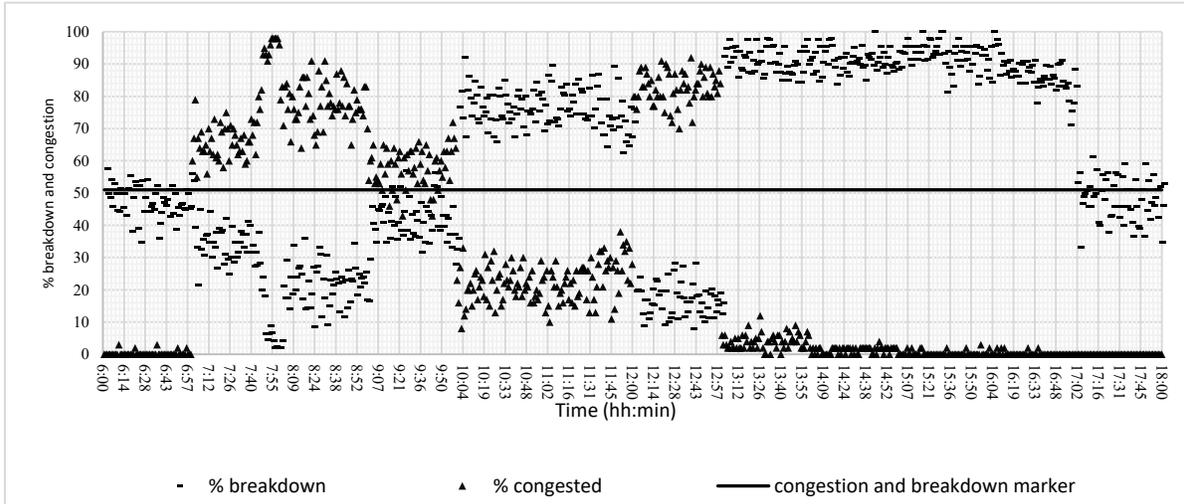


Figure 5 Comparing breakdown with congestion (January-February weekdays)

In the August data and when the percentile density between the congested and free flow (PDCF), is above 50%, it was used to identify the breakdown also shown in Figure 6. However, to briefly check this breakdown, the congested state that was derived using the threshold method was compared with the PDCF as shown in Figure 6. In comparing the percentage of congestion with the percentile breakdown, the focus was placed on the earlier congestion times derived using the threshold method. This approach aimed to identify the activity of breakdowns around those times. When the percentage of

breakdowns is low, not exceeding the 50% mark, it indicates a low probability of breakdowns. The following times were confirmed that breakdown occurred as shown in the Figure 7 above showed that only 09:05 am, 9:35am and 09:42 am showed both breakdown and congestion. This further shows the reliability of the methodology as it aligns with studies that mentioned that breakdown and congestion may not occur at the same time [2]. The comparison of the congestion period with the PDCF (Observed breakdown) is further shown in a graphical form in Figure 6.

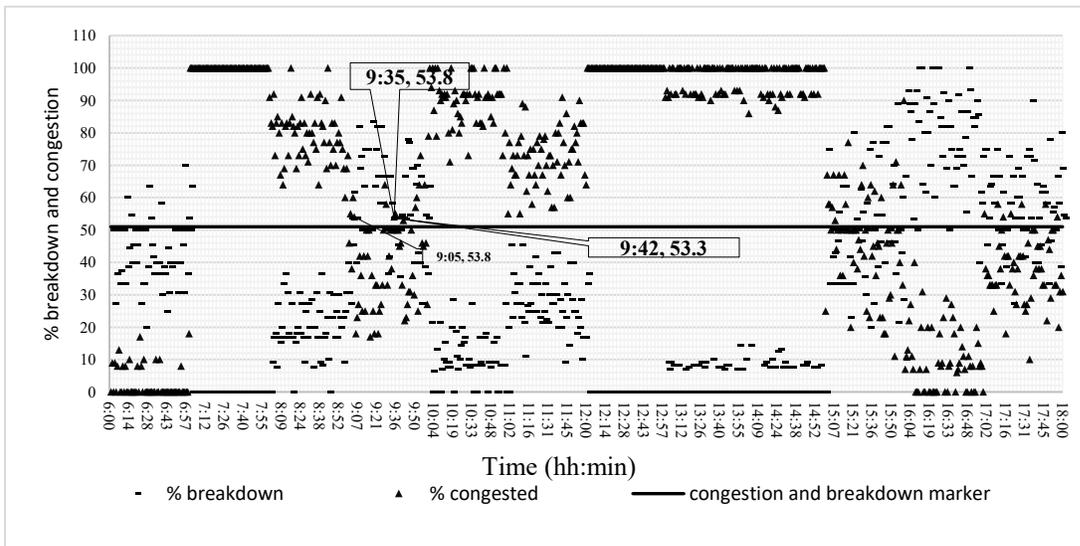


Figure 6 Comparing threshold congestion with PDCF (August weekdays)

**3.4 Breakdown modeling with Fuzzy Logic Control and Trapezoidal Membership**

The details of this procedure have been mentioned in section 2.3.1 of this paper. The fuzzy logic is used to cater for the gradual transition in traffic states. This gradual transition stems from the fact that traffic density and congestion do not abruptly switch

from “free flow” to “congested flow” at a fixed threshold. The degrees of membership (0 to 1) represent the extent of these high or low values. The fuzzy logic used in this manner helps to ensure accuracy, simplicity and interpretability of traffic flow systems. The Fuzzy Logic defined membership functions is shown in equations (1) to (3):

Membership Functions

For  $K_c$  (High Density Range):

$$\mu_{HD} K_c(x) = \begin{cases} 0 & x < 50 \\ \frac{x - 50}{50} & 50 \leq x < 100 \\ 1 & x \geq 100 \end{cases} \tag{1}$$

For  $K_f$  (Low Density Range):

$$\mu_{LD} K_f(x) = \begin{cases} 1 & x \leq 0 \\ \frac{50-x}{50} & 0 < x < 50 \\ 0 & x \geq 50 \end{cases} \tag{2}$$

For  $C_t$  (High congestion Range):

$$\mu_{HDC_t}(x) = \begin{cases} 0 & x < 50 \\ \frac{x - 50}{50} & 50 \leq x < 100 \\ 1 & x \geq 100 \end{cases} \tag{3}$$

Using these membership functions the combined breakdown equation for breakdown using fuzzy logic operators (min for AND):

$$Breakdown(t, \kappa) = \begin{cases} 1 & \text{if } (m = 0) \wedge [(\kappa < 50 \wedge \mu_{HD}(C_t) = 1) \vee \\ & (\kappa > 50 \wedge [t \in T_1 \wedge \mu_{HD} K_c \geq 0.5 \wedge \mu_{HD}(C_t) = 1 \vee \\ & t \in T_2 \wedge \mu_{HD}(C_t) > 1])] \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Where:

$t$  = time of day (in 24 – hour format)

$T_1$  = morning interval [06:00, 09:00]

$T_2$  = Daytime interval [09:00, 18:00]

$\kappa$  = Flow level ( $\kappa = k_f$  (%))

$m$  = minimum of traffic parameters:

$(m) = \min(\mu_{HD}k_c, \mu_{LD}k_f, \mu_{HDC_t})=0$

The crisps decision rules follows the pattern of traffic which stipulates that traffic during the beginning of the year is less dense and increases due to increase of activities towards the middle of the year [36,37]. This works follows such works and assumes to set the crisps decision rules for traffic in the study route. Crisp decision rule ensures all conditions of high density at congested speed, high density at free flow and low congestion are met simultaneously for breakdown to occur. The crisp decision rule to ascertain the occurrence of traffic breakdown is shown in equations (5), (6), and (7).

**Rule 1: low-flow regime ( $\kappa < 50$ )**

$$[if (m = 0) \wedge \kappa < 50 \wedge \mu_{HD}(C_t) = 1 \wedge t \in (06:00, 18:00) \text{ then breakdown occurs. it fails otherwise}] \tag{5}$$

**Rule 2: high – flow morning ( $\kappa > 50 \wedge t \in T_1$ )**

$$[if (m = 0) \wedge \kappa > 50 \wedge t \in (06:00, 09:00) \wedge \mu_{HD}k_c \geq 0.5 \wedge \mu_{HD}(C_t) = 1 \text{ then breakdown occurs. it fails otherwise}] \tag{6}$$

**Rule 3: high – flow Daytime ( $\kappa > 50 \wedge t \in T_2$ )**

$$[if (m = 0) \wedge \kappa > 50 \wedge t \in (09:00, 18:00) \wedge \mu HD(C_t) = 1 \text{ then breakdown occurs. it fails otherwise}] \quad (7)$$

The  $K_f < 50$  was introduced in the breakdown equation to cater for periods with reduced critical density resulting from market activities along roadway, impacts of weather e.g. visibility, driver behaviour such as longer headway as explained in [38].  $\wedge \mu HD(k_c) \geq 0.5$  was restricted to am peaks in order to capture directional peaks in line with Caltrans PeMS bottleneck thresholds [39].

**3.5 Model formulation**

Equation (4) integrates critical system requirements into a unified fuzzy logic-based formulation. The model operates through the use of trapezoidal membership functions, breakdown indicators and crisp decision rules:

**1. Membership Function Definitions:**

- For parameter for  $k_c$ , density value below 50% represents stable flow and are excluded from the high-density category which implies that its membership is zero (0). The transition range  $50 \leq x < 100$  with membership  $\frac{x-50}{50}$  means that at  $x=50\%$  membership is 0 (transition starts), as  $x$  increases, membership rises linearly. This represents the gradual onset of congestion, where density increases degrading traffic stability progressively. At  $x \geq 100$ , the membership is 1, which represents full congestion and captures full breakdown conditions where flow instability is certain.
- The membership function  $\mu LD K_f(x)$  quantifies the degree to which a given free-flow density value  $x$  (in %) belongs to the “low density” fuzzy set. At  $x=0$ , it implies

$$B(k_c, k_f, C_t) = \min [\mu HD(k_c), \mu HD(k_f), \mu LD(C_t)] \quad (8)$$

**3.6 Model Validation**

The model is evaluated across two distinct datasets:

- Initial Testing: this was conducted in January and February to calibrate membership thresholds and verify baseline performance.
- Extended Validation: This process involved application to August data to assess robustness under seasonal variability.

low density and free flow conditions. At  $\frac{50-x}{50}$  membership decreases linearly with increased density, implying gradual degradation of free flow conditions with increased density. At  $x \geq 50\%$ , membership becomes 0 indicating breakdown conditions.

- The membership  $\mu HD C_t(x)$  quantifies the degree to which a given congested state  $x$  (%) belongs to the high-density fuzzy set for congested threshold. When  $x < 50\%$ , the membership is 0, which represents stable traffic conditions and congestions has not reached critical state. At  $50 \leq x < 100$  a transition region is defined with a membership  $\frac{x-50}{50}$ , at  $x=50\%$  membership is zero setting the beginning of critical congestion threshold and increases linearly as  $x$  increases to depict a progressive loss in traffic stability that tends towards gridlock. At  $x \geq 100$ , the membership becomes 1, representing complete congestion or jam density.

The initial breakdown indicator which is given by equation (8) combines these conditions using the fuzzy AND operator (min). the full breakdown is shown in equation in the equation (4) which combines the minimum requirement of initial breakdown of equation (8) and also considers daily and seasonal traffic flow variations. The equation (4) also considers drivers behavior in order to predict the occurrence of traffic breakdown thereby ensuring a good number of conditions are satisfied for breakdown to occur.

**3.7 Breakdown model for January to February**

The  $K_f$  and  $K_c$  were obtained by calculating the percent of the densities obtained in Table 2 as well as the congested state  $C_t$  obtained Figure 7 the membership functions  $\mu HD(K_c)$ ,  $\mu HD(K_f)$ ,  $\mu LD(C_t)$  were obtained from the equations (1) to (3). The crisp decision rules were obtained using equations (5), (6) and (7). The breakdown indicator in equation (4) combines the minimum membership functions, drivers’ behaviors and flow conditions to arrive at a breakdown state. The result is shown on Figure 8 to test the breakdown model.

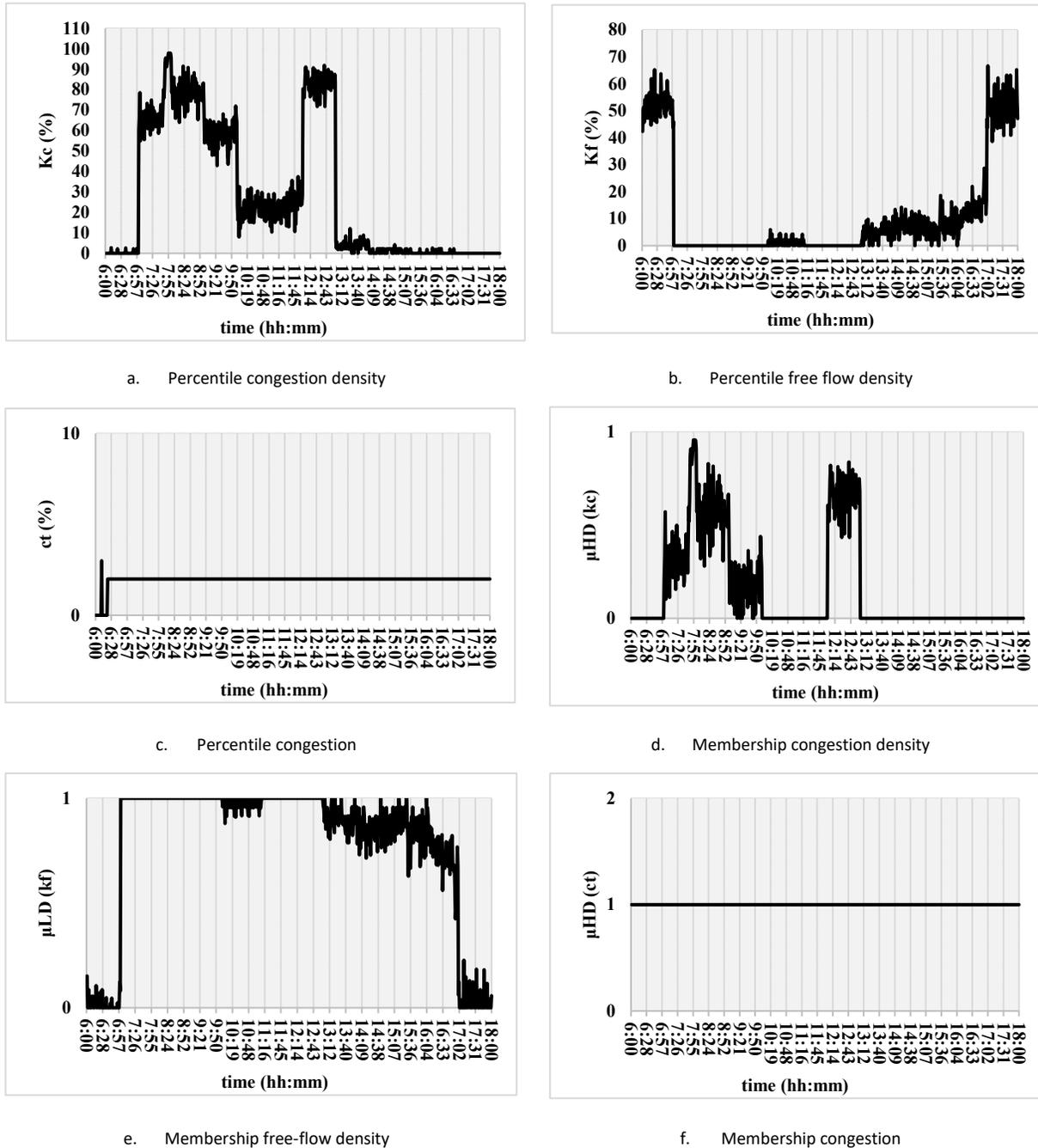


Figure 7 Derivation of fuzzy logic with trapezoidal membership functions for breakdown for January & February

Figure 8 shows the membership functions of the fuzzy logic with trapezoidal membership function. The result in Figure 8 was further used with a crisp decision rules equations (5), (6) and

(7) were applied to it to derive the breakdown of the road as shown in Figure 9.

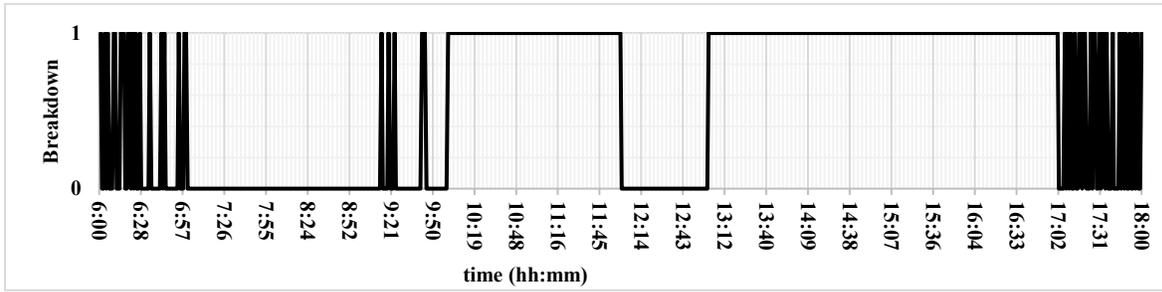
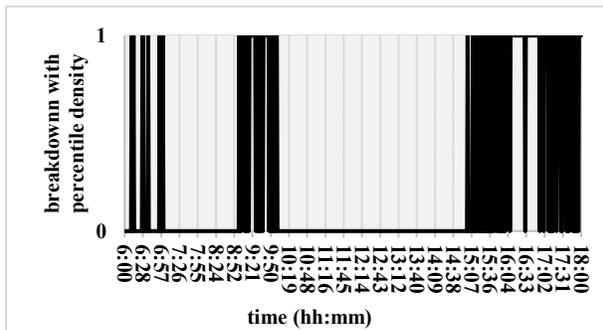


Figure 8 Fuzzy logic with trapezoidal membership function Breakdown result January to February

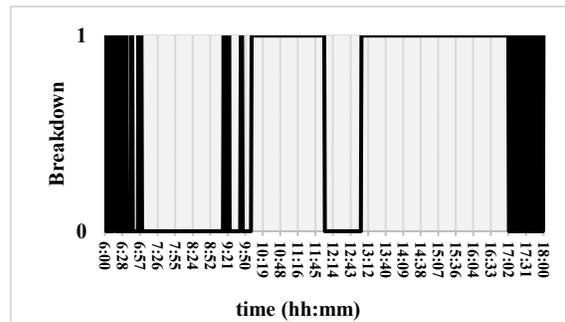
Figure 9 shows the testing of the breakdown model using the fuzzy logic model. The crisp rule as described in equations (5), (6) and (7) was set alongside the breakdown equation (4) to indicate a breakdown shown in Figure 9. In Figure 9, zero (0) was set to indicate no breakdown while 1 indicates breakdown. This was selected to facilitate the plot of the breakdown based on the time of occurrence. Through this process in Figure 9 a total of 410 breakdowns were found in the January / February dataset

A comparative analysis was conducted to evaluate the model's performance. The breakdown results obtained from the proposed model (BFLC) as derived in the Figure 9 were benchmarked against the breakdown from density percentile analysis values as shown on Figure 5, which were previously derived using the threshold method using (PDCF and compared

with the breakdown derived from the fuzzy logic model. In Figure 10 the 409 breakdown times identified by method without model (the threshold method) was compared with the 410 breakdown times identified by method with model (fuzzy logic model). The fuzzy logic model detected 6:11 am to be a breakdown time is the only difference between the two methods. It was also found that the fuzzy logic detected all the times that was detected by the percentile density method. Hence, it was concluded that in January and February, the model prediction of breakdown on this road is  $\frac{409 \times 100}{409} = 100\%$ . The comparison of the breakdown using the threshold method and the Fuzzy logic model is further shown in the Figure 10



(a)



(b)

Figure 9 Comparing breakdown of (a) using percentile densities with (b) using fuzzy model for Jan-Feb 2023 weekdays

The Figure 10 a, and b, was used to compare breakdowns captured by the fuzzy logic and threshold methods and shows a strong relationship between the two because they produce similar breakdown periods. To ascertain the model developed using the January and February data, it was applied to the August data. There is a strong relationship between the two methods, because they both indicate similar periods of breakdown.

To validate the model developed using the January and February data, it was applied to August data to test its workability.

### 3.8 Breakdown Model for August

K<sub>f</sub> and K<sub>c</sub> were obtained after taken the percentages of K<sub>ff</sub> and K<sub>cs</sub> in

Table 2 as well as the congested state C<sub>t</sub> obtained in Figure 7, the membership functions  $\mu_{HD}(K_c)$ ,  $\mu_{HD}(K_f)$ ,  $\mu_{LD}(C_t)$  were obtained from the equations (1) to (3). The crisp decision rules were obtained using equations (5), (6) and (7). The breakdown indicator in equation (8) uses the minimum of the membership functions to arrive at a breakdown state. The result is shown on Figure 11 to validate the breakdown model

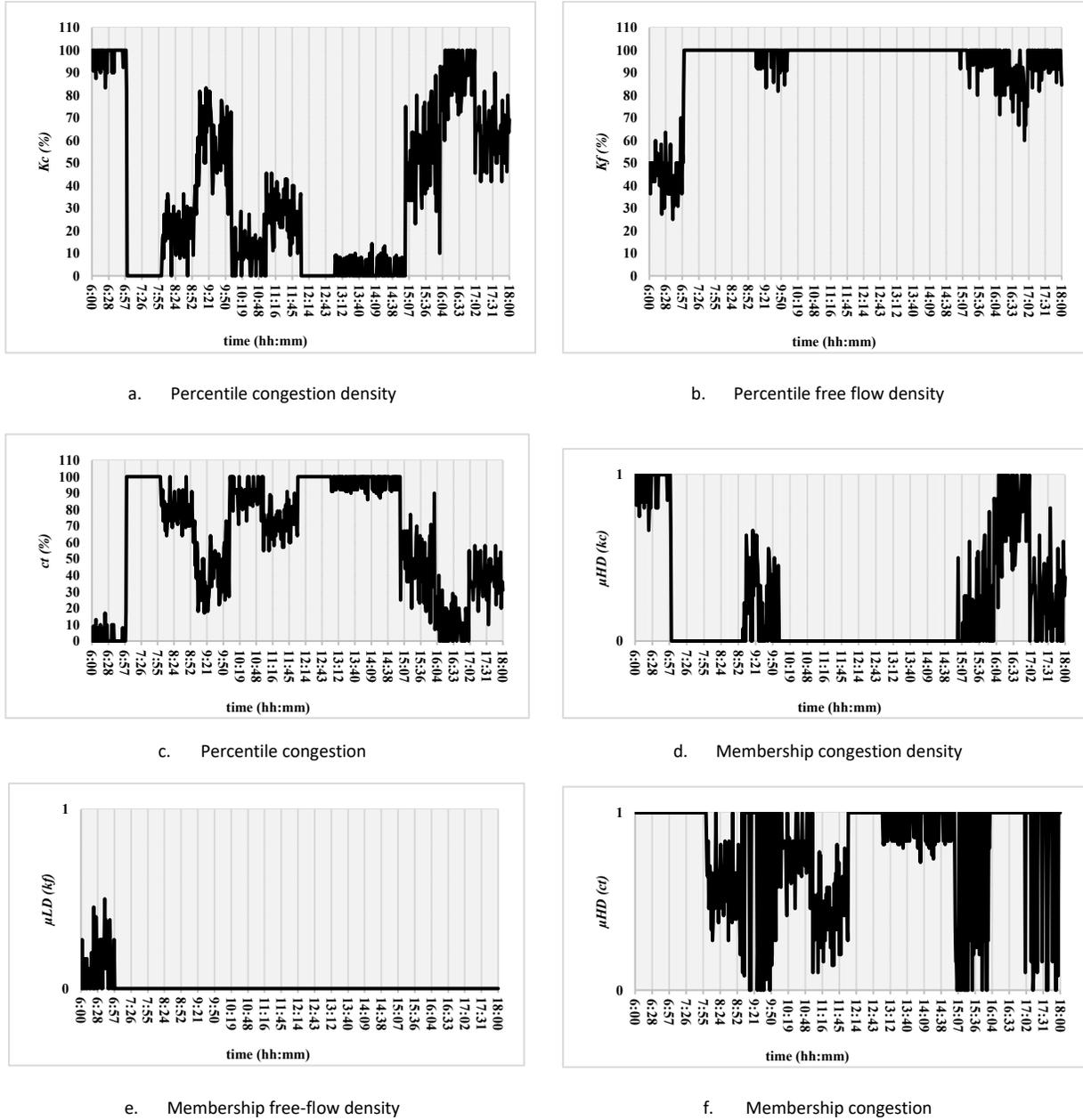


Figure 10 Derivation of fuzzy logic with trapezoidal membership functions for breakdown for August

The aim of producing the fuzzy membership for August 2023 as shown Figure 11 is to validate the model of the breakdown that was calibrated with the January February data set. The crisp rule as described in equations (5), (6) and (7) was set alongside the minimum value of  $\mu_{LD}(C_t)$ ,  $\mu_{HD}(k_f)$  and  $\mu_{HD}(k_c)$  in a breakdown equation (4) to indicate a breakdown shown in

Figure 12. In Figure 12, zero (0) was set to indicate no breakdown while 1 indicates breakdown. This was selected to facilitate the plot of the breakdown based on the time of occurrence. Through this process as presented in Figure 9 a total of 184 breakdowns were found in the August dataset.

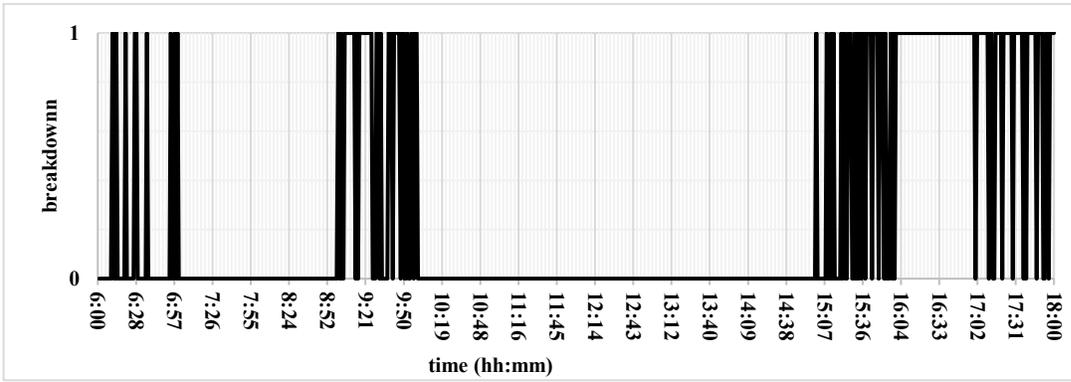
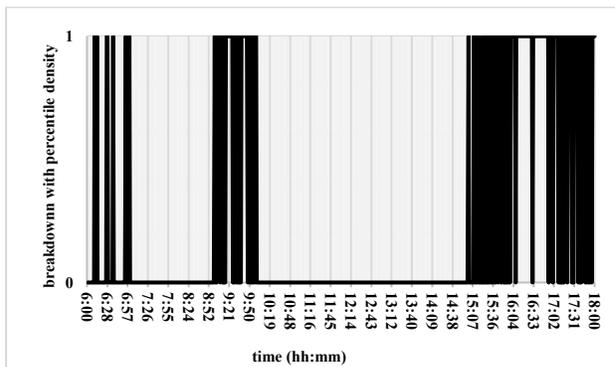


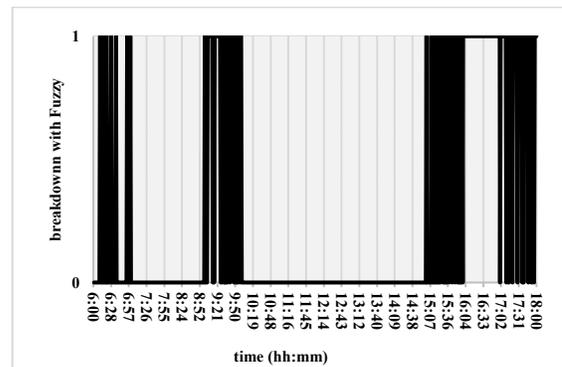
Figure 11 Fuzzy logic with trapezoidal membership function Breakdown result August

In evaluating the model's performance. The breakdown results obtained from the proposed model which was derived using the fuzzy logic modeling were benchmarked against the percentile

derived breakdown values, which were previously derived using the threshold method (PDCF) and compared as shown in Figure 12.



(a)



(b)

Figure 12 Comparing breakdown for (a) with percentile density method and (b) with fuzzy model for August 2023 weekdays

As illustrated in Figure 12, the threshold method identified 167 distinct breakdown instances across varying timestamps. For comparative evaluation, the fuzzy logic method was applied to the same dataset, detecting 184 breakdown events Figure 12. To assess alignment between the two approaches, overlapping breakdown timestamps were analyzed.

The temporal correlation between the methods revealed 164 concurrent breakdown instances detected by both techniques. Given that the threshold method captured 167 events and 164 of these overlapped with fuzzy logic results, the accuracy of the fuzzy logic model in replicating threshold-based detection was calculated as 98.20%:

To evaluate the average magnitude of discrepancy between the two breakdown identifications methods, the Root Mean Square of Error (RMSE) was calculated. The analysis used the breakdown earlier identified by the threshold method in

along with the fuzzy logic method derived for January/February as in Figure 8 and August in The Figure 12. The RMSE analysis for these periods is presented in Table 3.

Table 3 Root mean square of error for breakdown validation

|                    | Total sum of squared errors (TSSE) | Total error from two methods (TE) | Mean squared error (MSE)     | RMSE                        |
|--------------------|------------------------------------|-----------------------------------|------------------------------|-----------------------------|
| January / February | 96                                 | 289                               | $= \frac{96}{289} = 0.3322$  | $= \sqrt{0.3322} = 0.5764$  |
| August             | 23                                 | 720                               | $= \frac{23}{720} = 0.03194$ | $= \sqrt{0.03194} = 0.1787$ |

The breakdown which occurred in January/February was converted into minutes to allow for numerical analysis. The RMSE is about 0.5764 minutes, translating to 35 seconds, which indicates a high similarity between the two methods in detecting breakdown. The August analysis also showed that both methods are close in detecting breakdown because the two methods agree on approximately 97.1%

### 3.9 Analysis of Peak volume and Breakdown Correlations

This section focuses on the connection between the PHV and the road traffic breakdown occurrences. The analysis focuses on

data collected from January-February and August 2023 and involved calculating average hourly volume and PHV. The PHV was finally associated with the breakdown events.

### 3.9.1 Average Hourly Volume

The data collection was from 06:00 to 18:00; which implies the analysis was based on average hourly volumes and not daily estimates. Traffic volumes were calculated per minute and aggregated into 15-minute interval as shown in Figure 13.

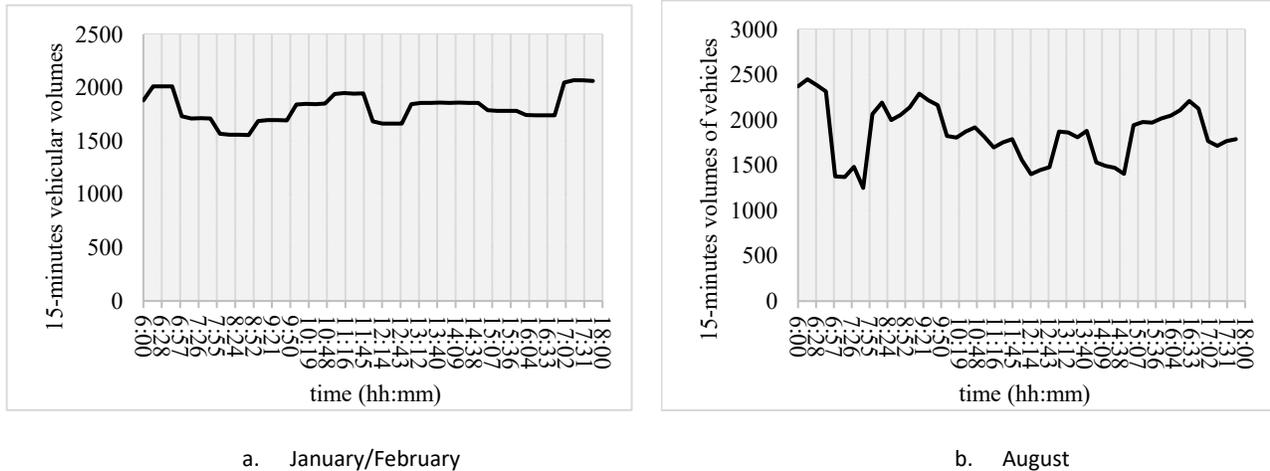


Figure 13 15-Minutes Vehicle volumes for January/February 2023

The January/February traffic pattern analysis Figure 14a shows distinct periods in conformity with commuter activity. The morning peak was observed in the early morning due to movement towards the city centre. The midday peak occurs around (11:00-12:00) caused by market activity. The evening peak was detected from 17:00 to 18:00 as a result of return of commuters. An almost similar pattern was analysed in the August data Figure 14b, with peaks occurring in the early morning and around 16:45, with more peaks identified at 08:30 and 09:30. The total hourly traffic volume (06:00 to 18:00) was calculated based on the sum of the hourly volumes. In Figure 14 A total of 87,003 vehicles per day were reckoned in January / February whereas, in August this volume was found to be 89,265 vehicles per day. This shows that there is an increase of 2,262 vehicles per day (2.6%), between the study periods.

### 3.9.1 Peak Hour Volume

The PHV was obtained from the 15-minute traffic volumes as in Figure 14 a, and b, as well as the calculations summarized in Table 4. This analysis was carried out for January / February and compared with the August periods.

The January /February analysis showed that the highest 15-minute volume occurred around 17:15 with 2,070 vehicles to arrive at a PHV of 8,280 vehicles per hour. To account for flow

variability, an adjusted PHV was derived as 7,618 vehicle per hour.

Using the August data, a higher maximum 15-minute volume amounting to 2,450 vehicles were found at an earlier time of 06:15, to produce a PHV of 9,800 vehicles per hour. As a result, the August data showed greater flow variability, due to the higher standard deviation and coefficient of variation it produced. Consequently, the PHF produced an adjusted PHV of 8,406 vehicles per hour in the August data.

The result shows an increase in the adjusted PHV from 7,618 veh/h in January/February to 8,406 veh/h in August 2023. This increase indicates a change in traffic patterns that could lead to more frequent traffic breakdowns on this roadway if the right measures are not taken.

The PHF values of 0.93 from the January/February analysis indicates the road traffic volume remained consistently high and uniform, during the peak hour. Conversely a PHF of 0.86 obtained in the August data analysis means that the congestion level and breakdown likelihood has increased as it indicates increased vehicular volumes that the road cannot handle with ease.

The high demand on this road, detected in this study through the PHF and the PHV confirms that the Keffi- Abuja motorway is susceptible to breakdown. This finding further supports the hybrid percentile density and fuzzy logic model which detected breakdown on this road.

Table 4 Peak hour volume calculations

| January/February  |  | August  |  |
|---|--|---|--|
| Highest 15-minute volume (vehicles)                     | 2070                                     | Highest 15-minute volume (vehicles)                     | 2450                                     |
| Peak hour volume (PHV) (veh/min)                        | $2070 \times 4 = 8280$                   | Peak hour volume (PHV) (veh/min)                        | $2450 \times 4 = 9800$                   |
| Standard deviation of volume of vehicles                | 138.5478                                 | Standard deviation of volume of vehicles                | 304.1686                                 |
| Mean  | 1804.413                                 | Mean  | 1834.027                                 |
| Coefficient of variation (CV) = standard deviation/mean | $= \frac{138.5478}{1804.413} = 0.076783$ | Coefficient of variation (CV) = standard deviation/mean | $= \frac{304.1686}{1834.027} = 0.165847$ |
| Peak hour factor (PHF) = $\frac{1}{1+cv}$               | $= \frac{1}{1+0.076783} = 0.928692$      | Peak hour factor (PHF) = $\frac{1}{1+cv}$               | $= \frac{1}{1+0.165847} = 0.857745$      |
| Adjusted peak hour volume = PHF x PHV (veh/hour)        | $= 0.928692 \times 8280 = 7618$          | Adjusted peak hour volume = PHF x PHV (veh/hour)        | $= 0.857745 \times 9800 = 8406$          |

The findings of this study underscore the effectiveness of combining density percentile analysis with fuzzy logic control for detecting traffic breakdowns. By analyzing the relationship between speed, density, and flow, the study identified specific time periods with a high probability of traffic breakdown. The threshold speed of 30.1 km/h for January-February and 38.8 km/h for August served as critical benchmarks for distinguishing between congested and free-flow states. The thresholds, combined with the speed density relationship, provided a reliable basis for breakdown detection, on the percentile density analysis platform.

The binary logic controller with trapezoidal membership function model, based on the formulated crisps decision rules of this paper, is useful in surmounting uncertainties such as drivers' behavior, fluctuations in traffic flows which are common with road traffic. This model of breakdown detection successfully considered and incorporated these uncertainties to study the transition of traffic from its free – flowing states until it gradually moves into a breakdown and congested state. The validation of the model was carried out by comparing its breakdown detection capabilities against the percentile density analysis method, achieving more than 98.8% accuracy across January-February and August data sets. This highlights the efficacy of this proposed hybrid approach for real-life traffic situations.

The breakdown model derived through this study can be used in traffic management and urban planning to adjust traffic signals, implement dynamic traffic systems, because it can identify specific periods of breakdown occurrence. Again, this model will also help with the accurate road improvement strategies due to its ability to predict traffic bottlenecks which can impede road capacity.

Summarily, this hybrid model approach, developed in this study, provides the basic platform for detecting traffic breakdowns that is intended to improve traffic management in general.

#### 4.0 CONCLUSIONS

The hybrid approach used in this study successfully brought together components of the percentile density analysis into the fuzzy logic control with trapezoidal membership functions and crisps decision rules. The major focus of this study, which was

achieved, identified congestion patterns and breakdown periods using both the percentile density analysis as well as the fuzzy logic control in Masaka situated along the Keffi- Abuja motorway (Nigeria). The study identified congestion periods, developed speed density models to derive percentile densities of free and congested. This culminated in a traffic breakdown model validated with the percentile density analysis procedure.

The study contributed to developing traffic breakdown model which incorporated vital traffic uncertainties needed for a robust breakdown detection. The study is useful to transport authorities to design early interventions to curtail traffic congestion and improve the capacity of the road.

Despite the limitations of this study which revolves around generalizability and improved data collection techniques, it has provided the platform for future research in this field. Considering weekends traffic flow, improving the data collection strategies, and using more advanced modeling techniques will ensure the development of an improved traffic breakdown model. Generally, this research contribution is essentially on the increasing research on traffic flow characteristics and traffic modeling which are aimed at providing transportation stakeholders with the appropriate tools to foster a better sustainable transportation system.

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#### Conflicts of Interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper

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